

Denksport

Schach - 8-Damenproblem



8 Damen so platzieren, dass sich keine gegenseitig schlagen.

Wiederholung

Gesucht:

- lineare Funktion $f(x)$ (Gerade) durch $P(-4/7)$ und $Q(3/-2)$
- liegt $R(1/1,5)$ auf der Geraden?
- Nullstelle?
- Zeichnung/Graph der Funktion

$$k = \frac{\Delta y}{\Delta x} = \frac{7 - (-2)}{-4 - 3} = -\frac{9}{7}$$

$$f: y = kx + d$$

$$-2 = -\frac{9}{7} \cdot 3 + d$$

$$-2 = -\frac{27}{7} + d \quad | +\frac{27}{7}$$

$$-\frac{14}{7} + \frac{27}{7} = d$$

$$\frac{13}{7} = d$$

$$\Rightarrow y = -\frac{9}{7} \cdot x + \frac{13}{7}$$

$$0 = -\frac{9}{7} \cdot x + \frac{13}{7} \quad | +\frac{9}{7}x$$
$$+\frac{9}{7}x = \frac{13}{7} \quad | : \frac{9}{7}$$

$$\Leftarrow x = \frac{13}{7} \cdot \frac{7}{9} = \frac{13}{9} = 1\frac{4}{9} = 1,4\bar{4}$$

Nullstelle $y=0$

$$\underline{\underline{N(1,4\bar{4} | 0)}}$$

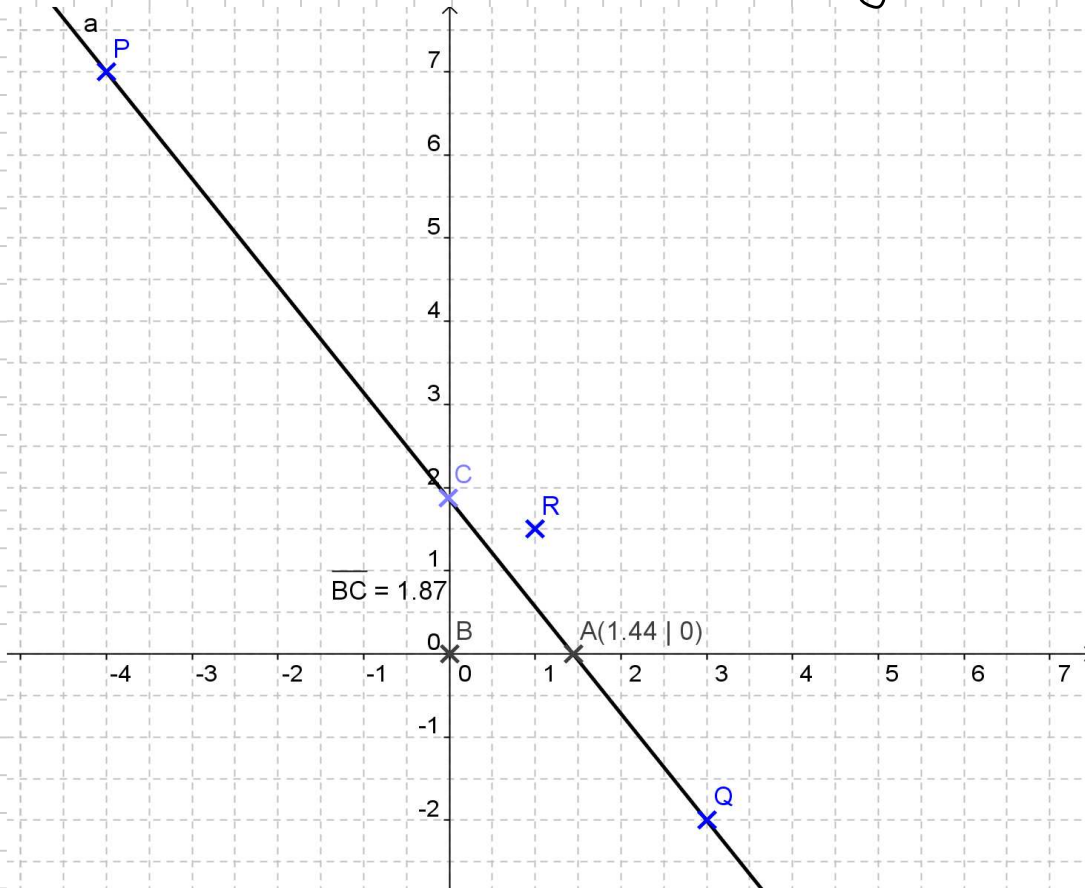
$$R \in f? \quad R(1|1,5)$$

$$y = \frac{k}{q} \cdot x + \frac{d}{q}$$
$$1,5 \neq -\frac{9}{7} \cdot 1 + \frac{13}{7}$$

$$1,5 \neq \frac{4}{7}$$

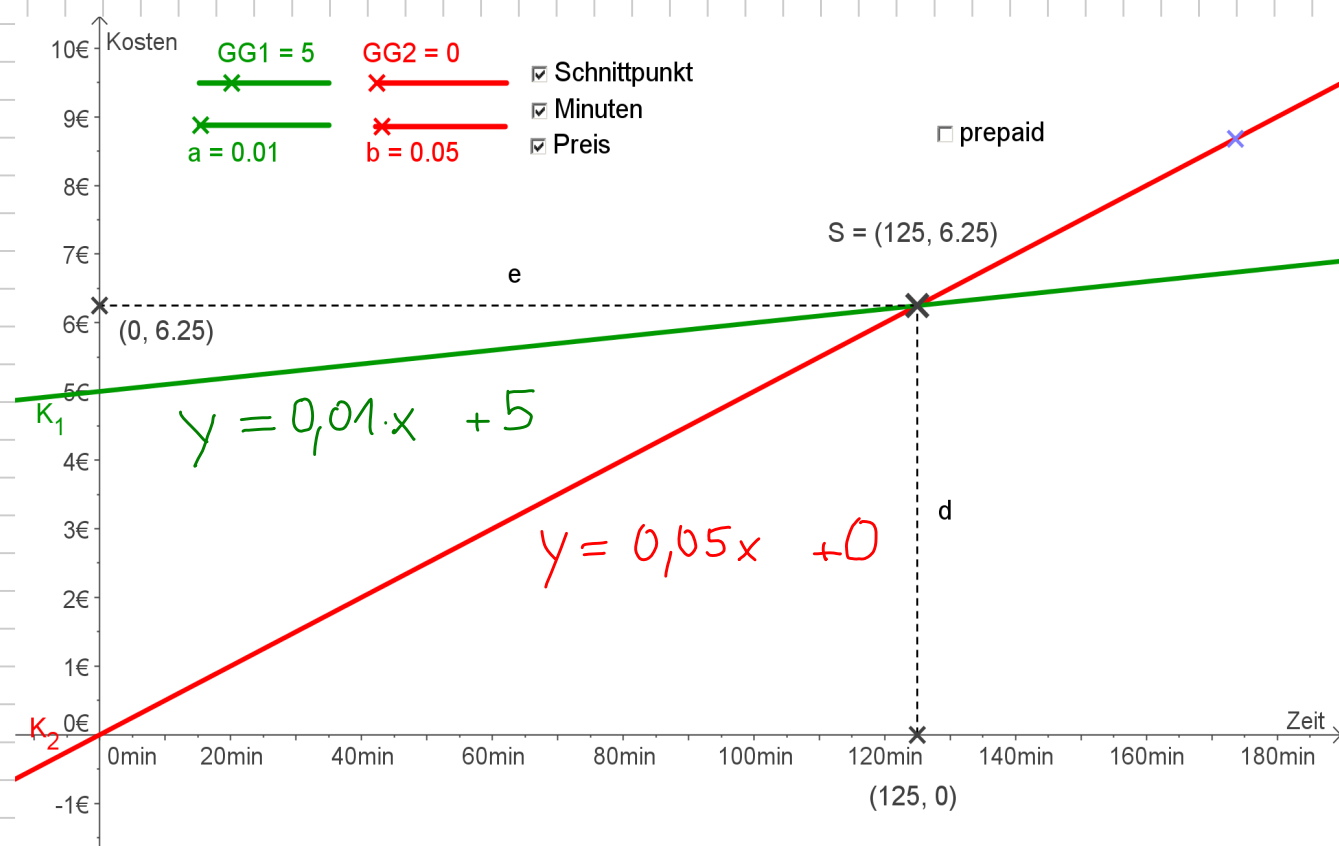
$$f.A. \Rightarrow R \notin f$$

R liegt nicht auf
Geraden



Lineare Gleichungssysteme

Gleichungssystem mit 2 Unbekannten



$$\text{I: } y = 0,01x + 5$$

x ... Minuten

$$\text{II: } y = 0,05x$$

y ... Kosten

$$y = y$$

$$0,01x + 5 = 0,05x$$

$$5 = 0,04x$$

$$125 = x$$

$$\Rightarrow y = 0,05 \cdot 125 = 6,25$$

$$S(125 | 6,25)$$

impl \rightarrow expl
 $-0,01x + y = 5$

S 2/9

$$\text{I: } 2x + 3y = 20$$

impl \rightarrow expl.

$$\text{II: } 3x - 4y = 13$$

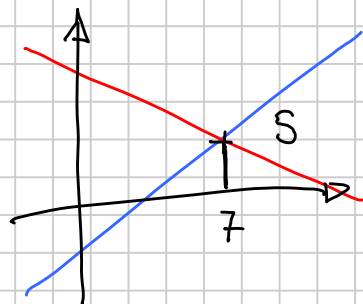
$$I: 3y = 20 - 2x = -2x + 20 \quad | :3$$

$$I: y = -\frac{2}{3}x + \frac{20}{3} \quad \text{expl}$$

$$II: (-4)y = +13 - 3x = -3x + 13 \quad | :(-4)$$

$$y = \frac{-3x + 13}{-4}$$

$$II: y = +\frac{3}{4}x - \frac{13}{4} \quad \text{expl.}$$



Gleichsetzungs-
verfahren

$$y = y$$

$$\frac{3}{4}x - \frac{13}{4} = -\frac{2}{3}x + \frac{20}{3} \quad | +\frac{2}{3}x \quad | +\frac{13}{4}$$

$$\frac{3}{4}x + \frac{2}{3}x = \frac{20}{3} + \frac{13}{4}$$

$$\frac{9x + 8x}{12} = \frac{80 + 39}{12} \quad | \cdot 12$$

$$17x = 119 \quad | :17$$

$$\underline{x = 7}$$

$$\Rightarrow II: y = +\frac{3}{4} \cdot 7 - \frac{13}{4}$$

$$\underline{y = \frac{8}{4} = 2}$$

S(7|2)

Einsetzverfahren

$$I: y = -\frac{2}{3}x + \frac{20}{3} \quad \text{expl}$$

$$II: 3x - 4y = 13 \quad \text{impl}$$

$$3x - 4\left(-\frac{2}{3}x + \frac{20}{3}\right) = 13$$

$$3x + \frac{8}{3}x - \frac{80}{3} = 13 \quad | +\frac{80}{3}$$

$$\frac{9}{3}x + \frac{8}{3}x = \frac{39}{3} + \frac{80}{3}$$

$$\frac{17}{3}x = \frac{119}{3} \quad | \cdot 3 | : 17$$

$$\underline{x = 7}$$

$$\Rightarrow y = \dots = 2$$

S(7|2)

Additionsverfahren

$$\text{I: } 2x + 3y = 20 \quad | \cdot 4 \quad \text{impl}$$

$$\text{II: } 3x - 4y = 13 \quad | \cdot 3 \quad \text{impl}$$

$$4 \cdot \text{I: } 8x + 12y = 80$$

$$3 \cdot \text{II: } 9x - 12y = 39 \quad] +$$

$$17x \quad / \quad = 119 \quad | : 17$$

$$\underline{x = 7}$$

$$\Rightarrow y = \dots = 2$$

S(7|2)

S 2/12

$$\text{I: } 2x + y = -4 \quad | \cdot 2$$

$$\text{II: } 4x - 2y = 16$$

$$2 \cdot \text{I: } 4x + 2y = -8$$

$$\text{II: } 4x - 2y = 16 \quad] -$$

$$/ \quad 4y = -24 \quad | : 4$$

$$\underline{y = -6}$$

$$\Rightarrow \text{I: } 2x + (-6) = -4 \quad | +6$$

$$2x = +2$$

$$\underline{x = 1}$$

S(1|-6)

$$L = \{(1|-6)\}$$

S 2/15

$$\text{B.7 I: } 13x - 15y = -19$$

$$\text{II: } 3x - 5y = -9 \quad | \cdot 3$$

$$C = \begin{pmatrix} 13 & -15 \\ 3 & -5 \end{pmatrix}$$

$$D = \begin{pmatrix} -19 \\ -9 \end{pmatrix} \quad \begin{matrix} 2 \\ 1 \end{matrix}$$

$$\begin{array}{r}
 \text{I: } 13x - 15y = -19 \\
 \text{II: } -9x + 15y = +27
 \end{array}
 \left. \vphantom{\begin{array}{r} \text{I: } 13x - 15y = -19 \\ \text{II: } -9x + 15y = +27 \end{array}} \right] -$$

$$\begin{array}{r}
 13x - 15y = -19 \\
 -9x + 15y = 27
 \end{array}$$

$$4x \quad / \quad = +8$$

$$\underline{x = 2}$$

$$\begin{aligned}
 \Rightarrow 3 \cdot 2 - 5 \cdot y &= -9 \\
 6 - 5y &= -9 \quad | -6 \\
 -5y &= -15 \quad | :(-5) \\
 \underline{y} &= 3
 \end{aligned}$$

$$S(2|3) \quad L = \{(2|3)\}$$

B.8)

$$\begin{array}{r}
 \text{I: } 8 - (3x - 2y) = 6y - (x + y) \\
 \text{II: } 4x - (x + 4y - 1) = 5 - 3(7x - 2y)
 \end{array}$$

$$\begin{array}{r}
 \text{I: } 8 - 3x + 2y = 6y - x - y \quad | -6y + x + y \\
 \text{II: } 4x - x - 4y + 1 = 5 - 21x + 6y \quad | -5 + 21x - 6y
 \end{array}$$

$$\text{I: } 8 - 2x - 3y = 0 \quad | \cdot 12$$

$$\text{II: } -4 + 24x - 10y = 0$$

$$\underline{\text{I: } 96 - 24x - 36y = 0} \quad] +$$

$$92 \quad / \quad -46y = 0$$

$$92 = 46y$$

$$2 = y$$

$$\Rightarrow 8 - 2x - 3 \cdot 2 = 0$$

$$-2x + 2 = 0$$

$$2 = 2x$$

$$1 = x$$

$$\Rightarrow L = \{(1|2)\}$$

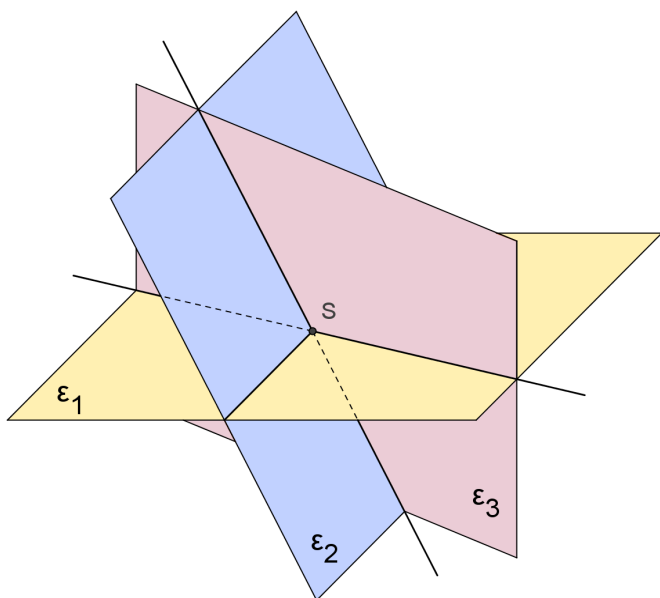
Gleichungssystem mit 3 Unbekannten

$$\begin{aligned} \text{I: } & 3x - 2y + z = 2 \\ \text{II: } & 4x + 3y - 2z = 4 \\ \text{III: } & x + 4y - 3z = 0 \end{aligned}$$

$$\begin{aligned} 2 \cdot \text{I: } & 6x - 4y + 2z = 4 \\ \text{II: } & 4x + 3y - 2z = 4 \end{aligned} \quad \left. \vphantom{\begin{aligned} 2 \cdot \text{I: } \\ \text{II: } \end{aligned}} \right\} +$$

$$\text{A) } 10x - y \quad / \quad = 8$$

Geometrisch:



$$\begin{aligned} 3 \cdot \text{I: } & 9x - 6y + 3z = 6 \\ \text{III: } & x + 4y - 3z = 0 \end{aligned} \quad \left. \vphantom{\begin{aligned} 3 \cdot \text{I: } \\ \text{III: } \end{aligned}} \right\} +$$

$$\begin{aligned} \text{B) } & 10x - 2y \quad / \quad = 6 \\ \text{A) } & -10x + y \quad = -8 \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{B) } \\ \text{A) } \end{aligned}} \right\} +$$

$$\quad / \quad -y \quad = -2$$

$$\quad \quad \quad y \quad = 2$$

$$\Rightarrow \text{A) } 10x - 2 = 8$$

$$\quad \quad \quad x = 1$$

$$\Rightarrow \text{I: } 3 \cdot 1 - 2 \cdot 2 + z = 2$$

$$3 - 4 + z = 2$$

$$\quad \quad \quad z = 3$$

$$S(1|2|3)$$

$$L = \{(1|2|3)\}$$

S 2|18

$$\begin{aligned} \text{I: } & 17x + 25y + 40z = 1054 \\ \text{II: } & 30x + 15y + 20z = 810 \\ \text{III: } & 24x + 19y + 30z = 928 \end{aligned}$$

$$A = \begin{pmatrix} 17 & 25 & 40 \\ 30 & 15 & 20 \\ 24 & 19 & 30 \end{pmatrix}$$

$$(-2) \cdot \text{II: } -60x - 30y - 40z = -1620$$

$$\text{A) } -43x - 5y \quad / \quad = -566$$

$$b = \begin{pmatrix} 1054 \\ 810 \\ 928 \end{pmatrix}$$

$$\begin{aligned} 3 \cdot \text{II} & 90x + 45y + 60z = 2430 \\ (-2) \cdot \text{III} & -48x - 38y - 60z = -1856 \end{aligned} \quad \left. \vphantom{\begin{aligned} 3 \cdot \text{II} \\ (-2) \cdot \text{III} \end{aligned}} \right\} +$$

$$A^{-1} \cdot b =$$

$$\begin{aligned} \text{B) } & 42x + 7y \quad / \quad = 574 \quad / :7 \\ & 6x + y \quad = 82 \quad / :5 \end{aligned}$$

$$\begin{aligned} 30x + 5y &= 410 \\ -13x &= -156 \quad / : (-13) \\ \underline{\underline{x}} &= \underline{\underline{12}} \end{aligned}$$

$$\begin{aligned} \Rightarrow 6 \cdot 12 + y &= 82 \quad / -72 \\ \underline{\underline{y}} &= \underline{\underline{10}} \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{II: } 30 \cdot 12 + 15 \cdot 10 + 20z &= 810 \\ 360 + 150 + 20z &= 810 \quad / -510 \\ 20z &= 300 \quad / : 20 \\ \underline{\underline{z}} &= \underline{\underline{15}} \end{aligned}$$

$$S(12|10|15)$$

$$L = \{(12|10|15)\}$$

1	→ 1	}	· 2	
2	→ 2			2^1
3	→ 4			2^2
4	→ 8			2^3
5	→ 16			2^4
⋮				

64 → 2^{63} 2 ^ 63

9,233 E18

Matrix

$$\begin{pmatrix} 1 & 4 & 7 \\ 2 & -2 & 8 \\ 3 & 1 & 9 \end{pmatrix}$$

3 x 3 - Matrix

$$\text{I: } 1x + y + z = 4$$

$$\text{II: } 2x - y + z = 3$$

$$\text{III: } 4x + 3y - 2z = 1$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 4 & 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$$

← Vektor

$$s^{-1} = \frac{1}{3}$$

$A \cdot x = b$

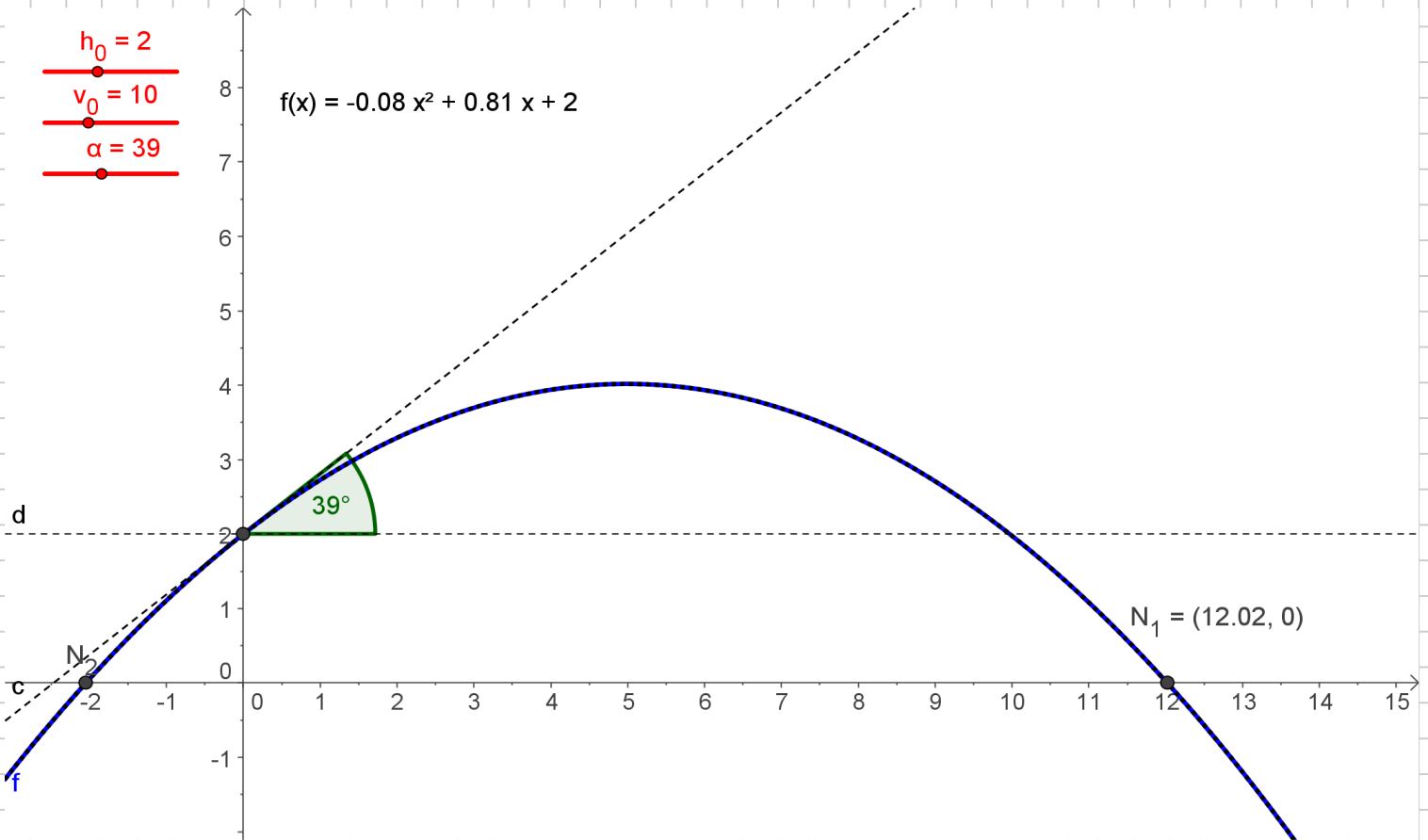
$\left[\begin{array}{l} 3 \\ 3 \cdot 3 \\ * = 3^{-1} \cdot 12 = 4 \end{array} \right] \cdot x = 12 \quad | \cdot 3^{-1}$

$\boxed{\vec{x} = A^{-1} \cdot b}$

$$[A]^{-1} * [B] = \left[\begin{array}{l} 0.764 \dots \\ 0.88 \dots \\ 2.35 \dots \end{array} \right] \begin{array}{l} x \\ y \\ z \end{array}$$

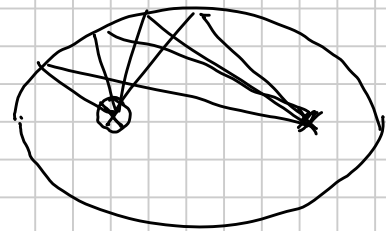
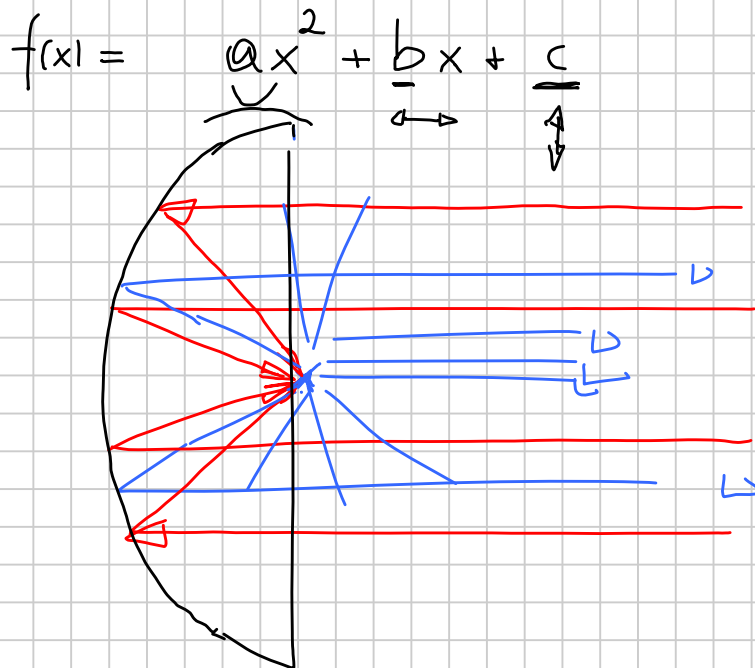
Die quadratische Funktion

Kugelstoßen - Kanonenschuss



S 2/63

Parabel



S 2/74 B. 2.3.c)

$$y = f(x) = ax^2 + bx + c$$

$$P(-1|-4) \in f(x) : a \cdot (-1)^2 + b \cdot (-1) + c = -4$$

$$Q(1|2) \in f(x) : a \cdot (1)^2 + b \cdot (1) + c = 2$$

$$R(2|11) \in f(x) : a \cdot 2^2 + b \cdot 2 + c = 11$$

$$\text{I: } 1a - 1b + c = -4$$

$$\text{II: } 1a + 1b + c = 2$$

$$\text{III: } 4a + 2b + c = 11$$

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{pmatrix} \quad b = \begin{pmatrix} -4 \\ 2 \\ 11 \end{pmatrix}$$

\Rightarrow TI 82

$$A^{-1} * b = 2 = a$$

$$3 = b$$

$$-3 = c$$

$$\Rightarrow \underline{\underline{f(x) = 2 \cdot x^2 + 3 \cdot x - 3}}$$