

Gleichungen + Funktionen

Notiztitel

29.10.2009

Denksport

$$a = b$$

$$/(a-b) = 0$$

$$3 = 5 / \cdot 0$$

$$0 = 0$$

$$a(a-b) = b(a-b)$$

$$a^2 - ab = \frac{ba}{ab} - b^2$$

$$|-ab| + b^2$$

$$a^2 + b^2 - 2ab = 0$$

$$a^2 - 2ab + b^2 = 0$$

$$(a-b)^2 = 0 \quad / \sqrt{\quad}$$

$$(a-b) = 0 \quad / (a-b) = 0$$

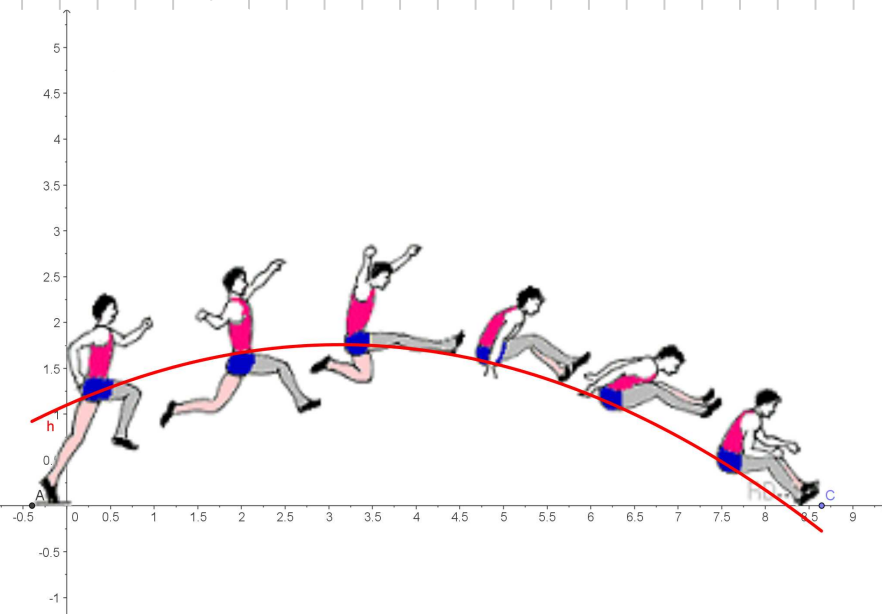


$$1 = 0$$

?

Wiederholung

Weit- und Sprung



$$B(0,36 | 1,24)$$

$$D(2,02 | 1,67)$$

$$E(3,37 | 1,75)$$

$$f(x) = \underline{a}x^2 + \underline{b}x + \underline{c}$$

$$\text{I: } f(0,36) = 1,24 = a \cdot 0,36^2 + b \cdot 0,36 + c$$

$$\text{II: } f(2,02) = 1,67 = a \cdot 2,02^2 + b \cdot 2,02 + c$$

$$\text{III: } f(3,37) = 1,75 = a \cdot 3,37^2 + b \cdot 3,37 + c$$

$$A = \begin{pmatrix} 0,36^2 & 0,36 & 1 \\ 2,02^2 & 2,02 & 1 \\ 3,37^2 & 3,37 & 1 \end{pmatrix}$$

$$b = \begin{pmatrix} 1,24 \\ 1,67 \\ 1,75 \end{pmatrix}$$

$$A \cdot x = b$$

$$x = A^{-1} \cdot b$$

$$x = A^{-1} \cdot b = \begin{array}{rcl} -0,0663 & = & a \\ 0,4169 & = & b \\ 1,0984 & = & c \end{array}$$

$$\Rightarrow f(x) = -0,0663 \cdot x^2 + 0,4169x + 1,0984$$

$$? \text{ Werte } f(x) = y = 0 = -0,0663x^2 + 0,4169x + 1,0984$$

→ Nullstelle finden / Quadratische Gleichungen

S. 2/20

$$ax^2 + bx + c = 0$$

1. Typ

$$ax^2 + c = 0$$

$$\begin{array}{rcl} \bullet & 2x^2 - 18 & = 0 \quad | +18 \\ & 2x^2 & = 18 \quad | :2 \\ & x^2 & = 9 \end{array}$$

$$x_1 = 3$$

$$x_2 = -3$$

$$(-3)^2 = 9$$

$$L = \{-3; 3\}$$

$$\begin{array}{rcl} \bullet & 5x^2 - 15 & = 0 \quad | +15 \\ & 5x^2 & = 15 \quad | :5 \\ & x^2 & = 3 \end{array}$$

$$x_1 = +\sqrt{3}$$

$$x_2 = -\sqrt{3} \approx -1,732 \dots$$

$$\begin{array}{rcl} \bullet & 2x^2 + 10 & = 0 \quad | -10 \\ & 2x^2 & = -10 \\ & x^2 & = -5 \end{array}$$

TR NONREAL

$$x \notin \mathbb{R} \quad [\in \mathbb{C}]$$

$$L = \{ \} \quad (\text{in } \mathbb{R})$$

2. Typ

$$ax^2 + bx = 0$$

$$\bullet \quad 5x^2 - 80x = 0$$

$$5x \cdot (x - 16) = 0$$

$$5x^2 = 80x \quad | :5x$$

$$\underline{\underline{x = 16}}$$

PNS

$$5x = 0 \quad | :5 \quad \text{oder}$$

$$x - 16 = 0 \quad | +16$$

$$\underline{\underline{x_1 = 0}}$$

$$\underline{\underline{x_2 = 16}}$$

$$L = \{0, 16\}$$

$$\bullet \quad 60x - 15x^2 = 0$$

$$15x(4 - x) = 0$$

$$15x = 0$$

$$\text{oder } 4 - x = 0$$

$$\underline{\underline{x_1 = 0}}$$

$$\underline{\underline{4 = x_2}}$$

$$L = \{0, 4\}$$

3. Typ

$$ax^2 + bx + c = 0 \quad | :a$$

3a

$$x^2 + px + q = 0$$

$$\bullet \quad x^2 + 4x + 4 = 0$$

$$(x + 2)^2 = 0$$

$$(x + 2)(x + 2) = 0$$

$$x_1 = -2$$

$$x_2 = -2$$

$$L = \{-2^{(2)}\}$$

Doppelösung

$$\bullet \quad \underline{x^2 + 4x} + 1 = 0$$

$$(\underline{x + 2})^2 - 4 + 1 = 0$$

$$(\underline{x + 2})^2 - 3 = 0$$

$$(\underline{x + 2})^2 = 3 \quad | \sqrt{\quad}$$

$$x^2 = 16$$

$$x = \pm 4$$

$$\begin{aligned}
 x+2 &= \pm \sqrt{3} \quad | -2 \\
 2x_1 &= -2 \pm \sqrt{3} \\
 x_1 &= -2 + \sqrt{3} = -0,2679\dots \\
 x_2 &= -2 - \sqrt{3} = -3,732\dots
 \end{aligned}$$

$$\bullet \quad \underline{x^2 + px + q = 0}$$

$$\left(x + \frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 + q = 0 \quad \left| +\left(\frac{p}{2}\right)^2 - q \right.$$

$$\left(x + \frac{p}{2}\right)^2 = \left(\frac{p}{2}\right)^2 - q \quad \left| \sqrt{\quad} \right.$$

$$x + \frac{p}{2} = \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \quad \left| -\frac{p}{2} \right.$$

pq-Formel

$$2x_1 = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$\Rightarrow \quad x^2 + \underline{5}x - \underline{4} = 0$$

$$p = 5$$

$$q = -4$$

$$2x_1 = -\frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - (-4)}$$

$$2x_1 = -2,5 \pm \sqrt{6,25 + 4}$$

$$2x_1 = -2,5 \pm \sqrt{10,25}$$

$$2x_1 = -2,5 \pm 3,2015\dots$$

$$\left\{ \begin{array}{l} \underline{x_1 = 0,7015} \\ \underline{x_2 = -5,7015} \end{array} \right.$$

$$\bullet \quad x^2 + x + 5 = 0$$

$$p = 1$$

$$q = 5$$

$$2x_1 = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - 5}$$

$$2x_1 = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 5}$$

$$2x_1 = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 5}$$

$${}_2x_1 = -\frac{1}{2} \pm \sqrt{-4,75} \quad \text{Diskriminante} < 0$$

$$L = \{ \}$$



3b

$$ax^2 + bx + c = 0$$

$$\cdot \underline{2}x^2 + \underline{3}x - \underline{20} = 0 \quad \Rightarrow \quad x^2 + \frac{3}{2}x - 10 = 0$$

\Rightarrow

ABC-FORMEL

$${}_2x_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$${}_2x_1 = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-20)}}{2 \cdot 2}$$

$${}_2x_1 = \frac{-3 \pm \sqrt{9 + 160}}{4}$$

$${}_2x_1 = \frac{-3 \pm \sqrt{169}}{4} = \frac{-3 \pm 13}{4}$$

$$x_1 = 2,5 \quad x_2 = -4$$

$$L = \{-4; 2,5\}$$

$$\begin{aligned} \text{Probe: } 2 \cdot (-4)^2 + 3 \cdot (-4) - 20 & \stackrel{!}{=} 0 \\ 32 - 12 - 20 & \stackrel{!}{=} 0 \end{aligned}$$

$$\cdot 4x^2 + x + 10 = 0$$

$${}_2x_1 = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 4 \cdot 10}}{2 \cdot 4}$$

$${}_2x_1 = \frac{-1 \pm \sqrt{-159}}{8} \quad \text{neg.}$$

$$L = \{ \}$$

T1 82

ABC-FORMEL

$$x_1 =$$

$$x_2 =$$



Satz von VIETA

$$(x+2)(x+3)$$

$$x^2 + 2x + 3x + 6$$

$$x^2 + 5x + 6$$

$$(x-2)(x-4) = 0$$

$$x_1 = 2 \quad x_2 = 4$$

$$(x-x_1)(x-x_2) = 0$$

Linearfaktoren

$$\text{III. } (x-x_1)(x-x_2) = x^2 + px + q = 0$$

$$x^2 - x_1 \cdot x - x_2 \cdot x + x_1 \cdot x_2 = 0$$

$$x^2 - (x_1 + x_2)x + x_1 \cdot x_2 = 0$$

I. $-(x_1 + x_2) = +p$

II. $x_1 \cdot x_2 = q$

$x_1 \quad x_2$ Lösung

$$\bullet \quad x^2 + 15x + 56 = 0$$

$$(x - (-7)) \cdot (x - (-8))$$

$$(x+7) \cdot (x+8)$$

$$\bullet \quad x^2 - 1x - 6 = 0$$

$$= (x-3) \cdot (x+2)$$

1,56

7,8

2,28

4,14

$$x_1 \quad x_2$$

$$\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix}$$

$$\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix}$$

0,5 ; -12

$$\begin{matrix} 1, -6 \\ 2, -3 \\ 3, -2 \\ 6, -1 \end{matrix}$$

$x_1 = 3 \quad x_2 = -2$

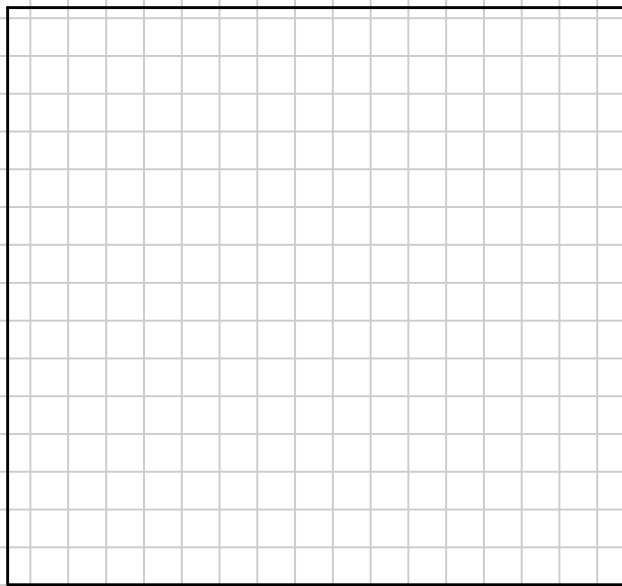
S 2/25

$$\bullet \quad x^2 + 6x - 7 = 0$$

$$x_1 = 1 \quad x_2 = -7$$

$$\Rightarrow \begin{matrix} 1, -7 \\ 7, -1 \end{matrix}$$

$$x^2 + 6x - 7 = (x-1) \cdot (x+7)$$



Funktionen schneiden

S 2/74

$$f: y = x^2 + 2x - 3$$

$$g: y = -3x + 5$$

$$\begin{array}{l} x^2 + 2x - 3 = -3x + 5 \quad | +3x | -5 \\ x^2 + 5x - 8 = 0 \end{array}$$

$$A = 1$$

$$B = 5$$

$$C = -8$$

T182

$$x_1 = 1,274... \quad \boxed{X}$$

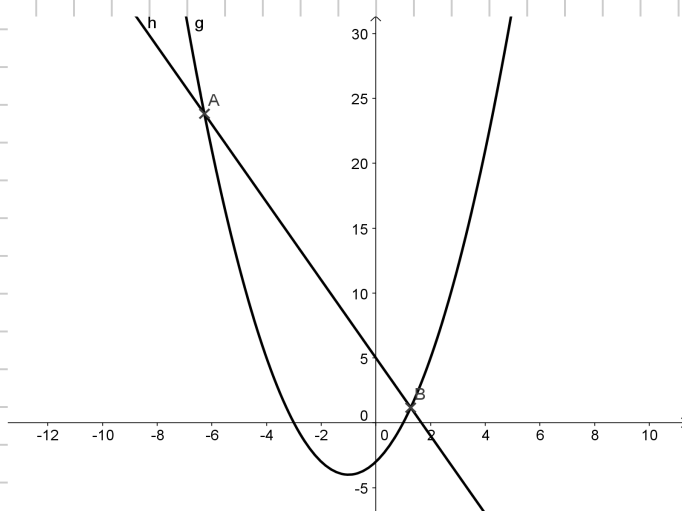
$$x_2 = -6,274... \quad \boxed{Y}$$

$$y_1 = -3 \cdot \boxed{X} + 5 = 1,17$$

$$S_1(1,274 | 1,17)$$

$$y_2 = -3 \cdot \boxed{Y} + 5 = 23,8$$

$$S_2(-6,274 | 23,8)$$



S 2/75

2.5c

$$f(x) = y = -\frac{3}{4}x^2 - 6x + 6$$

$$g(x) = y = -x^2 - 6x + 4$$

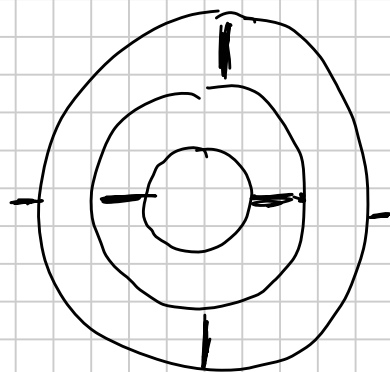
$$f \cap g : -\frac{3}{4}x^2 - 6x + 6 = -x^2 - 6x + 4 \quad | +x^2 \quad | +6x \quad | -4$$

$$\frac{1}{4}x^2 + 2 = 0$$

$$\frac{1}{4}x^2 = -2$$

$$x^2 = -8 \quad \downarrow$$

\Rightarrow keine Schnittpunkte



Gleichungen / Funktionen höheren Grades

$$③ -2x^2 - 11x + 12 = 0$$

$$ax^3 + bx^2 + cx + d = 0$$

↳ Formeln von Cardano



~~$$\left(\sqrt[3]{\frac{3a\left(\frac{-b}{3a}\right)^2 \frac{2b}{3a} + c}{27}} + \frac{b\left(\frac{-b}{3a}\right)^2 + a\left(\frac{-b}{3a}\right) - \frac{cb}{3a} + d}{4a^2} \right)^{\frac{1}{3}} + \left(\sqrt[3]{\frac{3a\left(\frac{-b}{3a}\right)^2 \frac{2b}{3a} + c}{27}} - \frac{b\left(\frac{-b}{3a}\right)^2 + a\left(\frac{-b}{3a}\right) - \frac{cb}{3a} + d}{4a^2} \right)^{\frac{1}{3}} - \frac{b}{3a}$$~~

$$x^3 - 2x^2 - 11x + 12 = 0$$

$$= (x - x_1) \cdot (x - x_2) \cdot (x - x_3) = 0$$

$$\pm 1, 2, 3, 4, 6, 12$$

$x=1$ $1^3 - 2 \cdot 1^2 - 11 \cdot 1 + 12 \stackrel{?}{=} 0$
 $1 - 2 - 11 + 12 \stackrel{?}{=} 0$

$$x^3 - 2x^2 - 11x + 12 = \boxed{(x - x_1)} \cdot (x - x_2) \cdot (x - x_3) = 0$$

$$(x^3 - 2x^2 - 11x + 12) : (x - x_1) = \underbrace{(x - x_2)}_{\sim x^2} \cdot (x - x_3)$$

$$(x^3 - 2x^2 - 11x + 12) : (x - 1) = x^2 - x - 12$$

$$\begin{array}{r} -x^3 + x^2 \\ \hline \end{array}$$

$$\begin{array}{r} -x^2 - 11x \\ + x^2 + x \\ \hline \end{array}$$

$$\begin{array}{r} -12x + 12 \\ -12x + 12 \\ \hline \end{array}$$

$$\begin{array}{r} -12x + 12 \\ -12x + 12 \\ \hline \end{array}$$

OR

$$x^3 - 2x^2 - 11x + 12 = (x-1) \cdot (x^2 - x - 12) = 0$$

$$x-1=0 \quad \text{oder} \quad x^2 - x - 12 = 0$$

$$\underline{x_1 = 1}$$

→ ABC-Formel

$$A = 1$$

$$B = -1$$

$$C = -12$$

$$L = \{-3, 1, 4\}$$

$$x_2 = 4$$

$$x_3 = -3$$

S 2/75

$$\underline{y =} \begin{cases} 2,5x^3 + 2x^2 - 3x - 1,5 = 0 \\ 5x^3 + 4x^2 - 6x - 3 = 0 \end{cases} \quad / \cdot 2$$

$$\underline{x_1 = 1}$$

(aus TR-SOLVER)

$$\underline{\underline{x=1}} \quad 5 \cdot 1^3 + 4 \cdot 1^2 - 6 \cdot 1 - 3 = 0 \quad \checkmark$$

$$(5x^3 + 4x^2 - 6x - 3) : (x - \underline{1}) = 5x^2 + 9x + 3$$

$$\underline{-5x^3 + 5x^2}$$

$$/ \quad 9x^2 - 6x$$

$$\underline{-9x^2 + 9x}$$

$$/ \quad +3x - 3$$

$$\underline{+3x - 3}$$

OR

$$5x^2 + 9x + 3 = 0$$

→ TR: ABC-FORMEL

$$A = 5$$

$$B = 9$$

$$C = 3$$

$$\underline{\underline{x_2 = -0,44\dots}}$$

$$\underline{\underline{x_3 = -1,35\dots}}$$