

Gleichungen (Wurzel-/Exponential/Log)



Denksport

Wiederholung

$$f(x) = x^4 - 4,5x^3 - 6x^2 + 39,5x - 30$$

? Nullstellen

? Graph

$$x^4 - 4,5x^3 - 6x^2 + 39,5x - 30 = 0$$

(MATH 0 : SOLVER)

$$\underline{x=1}^? : 1^4 - 4,5 \cdot 1^3 - 6 \cdot 1^2 + 39,5 - 30 \stackrel{?}{=} 0 \checkmark$$

$$(x^4 - 4,5x^3 - 6x^2 + 39,5x - 30) : (x-1) = x^3 - 3,5x^2 - 9,5x + 30$$

$$\begin{array}{r} x^4 - 4,5x^3 - 6x^2 + 39,5x - 30 \\ -x^4 + - + - 30 \\ \hline - 3,5x^3 - 6x^2 + 39,5x - 30 \end{array}$$

$$\begin{array}{r} - 3,5x^3 - 6x^2 + 39,5x - 30 \\ + 3,5x^3 + 3,5x^2 - 3,5x^2 - 3,5x \\ \hline - 9,5x^2 + 39,5x - 30 \end{array}$$

$$\begin{array}{r} - 9,5x^2 + 39,5x - 30 \\ + 9,5x^2 + 9,5x - 9,5x - 30 \\ \hline 30x - 30 \end{array}$$

$$\begin{array}{r} 30x - 30 \\ - 30x + 30 \\ \hline 0 \end{array}$$

$$30x - 30$$

$$30x - 30$$

OR

$$x^3 - 3,5x^2 - 9,5x + 30 = 0$$

⇒ MATH 1 : SOLVER

Gleichung eingeben

$$\underline{x_2 = -3}$$

$$x^3 - 3,5x^2 - 9,5x + 30 : (x+3) = x^2 - 6,5x + 10$$

$$\begin{array}{r} x^3 - 3,5x^2 - 9,5x + 30 \\ -x^3 + 3x^2 \\ \hline -6,5x^2 - 9,5x + 30 \\ +6,5x^2 - 19,5x \\ \hline -10x + 30 \\ +10x + 30 \\ \hline 60 \end{array}$$

OR

$$x^2 - 6,5x + 10 = 0$$

⇒ ABC-Formel

$$A = 1 \quad x_3 = 4$$

$$B = -6,5 \quad x_4 = 2,5$$

$$C = 10$$

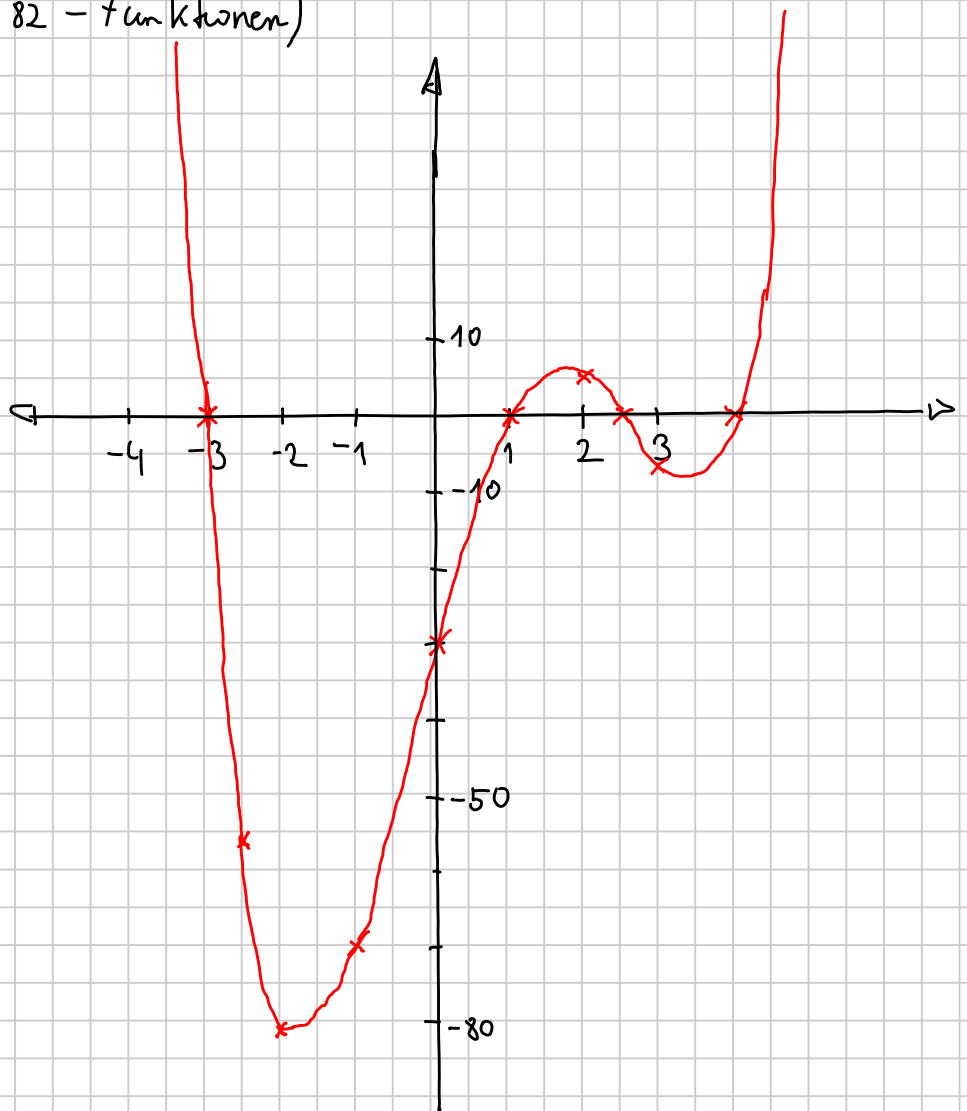
Nullstellen $N_1(1|0)$ $N_2(-3|0)$ $N_3(4|0)$ $N_4(2,5|0)$

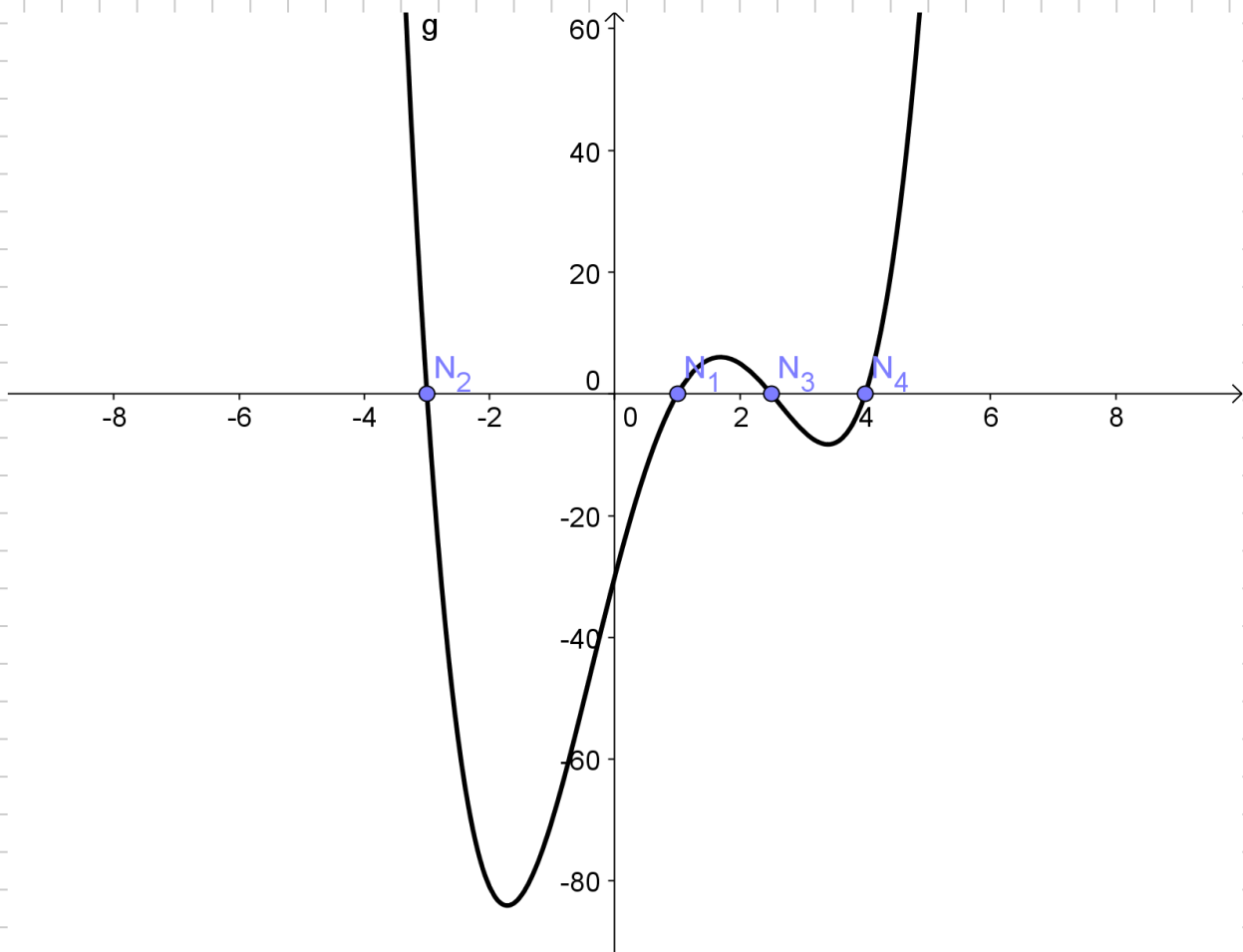
Graph (TI 82 - Funktionen)

→ Wertetabelle

$$I = [-6; 6]$$

x	y
-6	1785
-5	810
-4	260
-3	0
-2	-81
-1	-70
0	-30
1	0
2	5
3	-6
4	0
5	80
6	315





S 2/30 Bi-Quadratische Gleichungen

$$x^4 - 13x^2 + 36 = 0 \quad \Rightarrow \quad u^2 - 13u + 36 = 0$$

$$\boxed{u = x^2} \quad \Rightarrow \quad u^2 = x^4$$

ABC-Formel

$$A = 1 \mid B = -13 \mid C = 36$$

$$u_1 = 9$$

$$u_2 = 4$$

$$x^2 = 9$$

$$x^2 = 4$$

$$x_1 = -3$$

$$x_2 = +3$$

$$x_3 = -2$$

$$x_4 = +2$$

$$L = \{-3, -2, 2, 3\}$$

$$x^2 \cdot (x-3)(x+3) = 0$$

$$x^2 = 0$$

$$0^{(2)}$$

0 ist Doppellösung

S 2/32 Wurzelgleichungen

$$\sqrt{4x+1} + 7 = 2$$

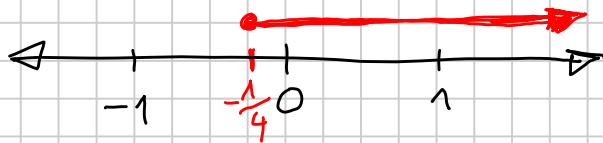
$$4x+1 \geq 0 \quad | -1$$

$$4x \geq -1 \quad | :4$$

$$x \geq -\frac{1}{4}$$

$$G = \mathbb{R}$$

$$D = \left[-\frac{1}{4}; \infty\right[= \{x \in \mathbb{R} \mid x \geq -\frac{1}{4}\}$$



$$\begin{aligned} -1 &\neq +1/2 \\ 1 &= 1 \end{aligned}$$

$$\sqrt{4x+1} + 7 = 2 \quad | -7$$

$$\sqrt{4x+1} = 5 \quad | \textcircled{2}$$

$$4x+1 = 25 \quad | -1$$

$$4x = 24$$

$$x = 6$$

Probe:

$$\sqrt{4 \cdot 6 + 1} + 7 = 2$$

$$5 + 7 = 2$$

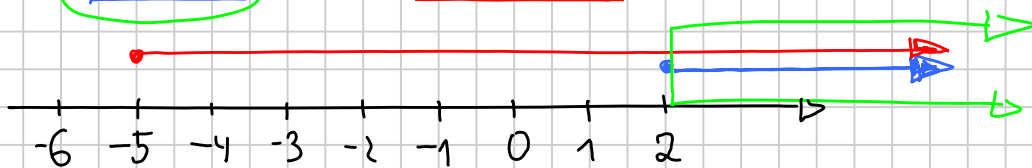
$$\Rightarrow L = \{\}$$

pos \neq neg
 $\Rightarrow L = \{\}$

Bsp. 2 $\sqrt{x-2} + \sqrt{x+5} = 7 \quad G = \mathbb{R}$

$$x-2 \geq 0 \quad \text{und} \quad x+5 \geq 0$$

$$\underline{x \geq 2} \quad \wedge \quad \underline{x \geq -5}$$



$$D = \{x \in \mathbb{R} \mid x \geq 2\}$$

$$\sqrt{x-2} + \sqrt{x+5} = 7 \quad | -\sqrt{x+5}$$

$$\sqrt{x-2} = 7 - \sqrt{x+5} \quad | \textcircled{2}$$

$$x-2 = 49 - 2 \cdot 7 \cdot \sqrt{x+5} + x+5 \quad | -54$$

$$+56 = +14 \cdot \sqrt{x+5} \quad | :14$$

$$4 = \sqrt{x+5} \quad | \textcircled{2}$$

$$= x+5$$

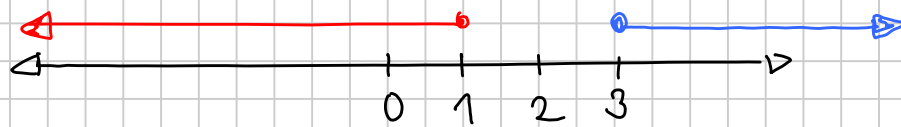
$$16$$

$$\frac{11}{3} = x$$

Probe: $\sqrt{11-2} + \sqrt{11+5} = 7$
 $3 + 4 = 7 \checkmark \Rightarrow L = \{11\}$

Bsp. $\sqrt{1-x} + \sqrt{x-3} = 4$

$$\begin{aligned} 1-x &\geq 0 & \textcircled{\wedge} & & x-3 &\geq 0 \\ 1 &\geq x & \wedge & & \underline{x} &\geq \underline{3} \\ \underline{x} &\leq \underline{1} & & & & \end{aligned}$$



$$\Rightarrow D = \{\} \Rightarrow L = \{\}$$

S. 2/34 5) $4\sqrt{x-8} \bigg| ^2$
 $16 \cdot (x-8)$

S 2/35 Exponentialgleichungen (vgl $x+1=3$)

Bsp $3^{x+5} - 2^x = 4^{x-1}$

a) Typ 1.

$$3^{x+1} = 3^4$$

$$\text{Basis}^{x+1} = \text{Basis}^4$$

$$\Rightarrow x+1 = 4$$

$$\underline{x = 3}$$

$$8 \cdot 2^x = 2^{3x-1}$$

$$2^3 \cdot 2^x = 2^{3x-1}$$

$$2^{3+x} = 2^{3x-1}$$

\Rightarrow

$$3+x = 3x-1 \quad | +1 | -x$$

$$4 = 2x$$

$$\underline{2 = x}$$

$$L = \{2\}$$

$$25 \cdot 5^{2x} = 125 : 5^{3x-1}$$

$$5^2 \cdot 5^{2x} = 5^3 : 5^{3x-1}$$

$$5^{2+2x} = 5^{3-(3x-1)}$$

$$\Rightarrow 2+2x = 3-3x+1$$

$$5x = 2$$

$$x = \frac{2}{5} = 0,4$$

$$L = \{0,4\}$$

b) Typ 2

$$2^{x+1} = 3^{x+1}$$

$$a^c = b^c$$

$$1 = 1$$

$$2^0 = 3^0$$

\Rightarrow

$$x+1 = 0$$

$$\underline{x = -1}$$

$$L = \{-1\}$$

$$125 \cdot 5^{3x} = 81 \cdot 3^{3x-1}$$

$$5^3 \cdot 5^{3x} = 3^4 \cdot 3^{3x-1}$$

$$5^{3+3x} = 3^{4+3x-1}$$

$$\Rightarrow 3+3x = 0$$

$$\underline{x = -1} \quad L = \{-1\}$$

c) Typ 3

$$2^x = 5 \cdot 3^x \quad / \ln$$

$$\ln \approx e$$

$$\lg \approx 10$$

$$(a^b)^c = a^{b \cdot c}$$

$$\ln 2^x = \ln 5 \cdot 3^x$$

$$x \cdot \ln 2 = \ln 5 + \ln 3^x$$

$$10^3 = 1000$$

$$x \cdot \ln 2 = \ln 5 + \underline{x \cdot \ln 3}$$

$$\Leftrightarrow {}^10\lg 1000 = 3$$

$$\uparrow \rightarrow \circ$$

$$\circ \rightarrow +$$

$$\therefore \rightarrow -$$

$$x \cdot \ln 2 - x \cdot \ln 3 = \ln 5$$

$$x \cdot (\ln 2 - \ln 3) = \ln 5$$

$$x = \frac{\ln 5}{\ln 2 - \ln 3}$$

$$\boxed{\ln(5) / (\ln(2) - \ln(3))}$$

$$x = -3,969\dots$$

$$L = \{-3,969\dots\}$$

Bsp.

$$3^{(x+1)} = 3^{(4)} \quad / \ln$$

$$\ln 3^{x+1} = \ln 3^4$$

$$(x+1) \cdot \cancel{\ln 3} = 4 \cdot \cancel{\ln 3} \quad / : \ln 3$$

$$\underline{\underline{x = 3}}$$

Bsp

$$2^{x+3} = 5^{x+3} \quad | \ln$$

$$(x+3) \cdot \ln 2 = (x+3) \cdot \ln 5$$

$$x \cdot \ln 2 + 3 \ln 2 = x \cdot \ln 5 + 3 \ln 5$$

$$x \cdot \ln 2 - x \cdot \ln 5 = 3 \ln 5 - 3 \ln 2$$

$$x (\ln 2 - \ln 5) = 3 (\ln 5 - \ln 2)$$

$$x = \frac{3 (\ln 5 - \ln 2)}{\ln 2 - \ln 5} = -3$$

S 2.38 Logarithmusgleichungen

$$p = p_0 \cdot \left(\frac{1}{2}\right)^{\frac{h}{5,5}}$$

1,01325

p_0 ... Luftdruck auf Meeresh.
 h ... Meereshöhe in km

$$p(885) = 1,01325 \cdot \left(\frac{1}{2}\right)^{\frac{885}{5,5}}$$

↑
Mount Everest

$$= 0,332$$

$$1,01325 \cdot 0,5^{\wedge}(885/5,5)$$

$$0,4 = 1,01325 \cdot 0,5^{\frac{h}{5,5}} \quad | : 1,01325$$

$$\frac{0,4}{1,01325} = 0,5^{\frac{h}{5,5}} \quad | \ln$$

$$\ln \frac{0,4}{1,01325} = \frac{h}{5,5} \cdot \ln 0,5 \quad | : \ln 0,5$$

$$5,5 \cdot \frac{\ln \frac{0,4}{1,01325}}{\ln 0,5} = \frac{h}{\cancel{5,5}} \cong 7,375 \text{ km}$$