Denksport

Wiederholung

S 4/14

B.1.10 $N(t) = N_0 \cdot a^t$

$2{,}5\,\% \rightarrow a = 1{,}025$

$N(t) = N_0 \cdot 1{,}025^t$

$N(t_1) = 1{,}5 \cdot \cancel{N_0} = \cancel{N_0} \cdot 1{,}025^t$

$1{,}5 = 1{,}025^t \quad | \ln$

$\ln 1{,}5 = t \cdot \ln 1{,}025$

$\frac{\ln 1{,}5}{\ln 1{,}025} = t$

$16{,}42 = t$

A: Nach 16,4 Jahren ist die Anzahl der Elefanten 1,5 mal so groß

b) $N_0 = 2000$

$N(4) = 2000 \cdot 1{,}025^4$

$N(4) = 2207$

$\big\downarrow -500$

$\overline{N(4)} = 1707$

$2000 = 1707 \cdot 1{,}025^t \quad | :1707$

$\ln \frac{2000}{1707} = \ln 1{,}025^t$

$\frac{\ln\left(\frac{2000}{1707}\right)}{\ln 1{,}025} = t$

$6{,}41\ldots = t$

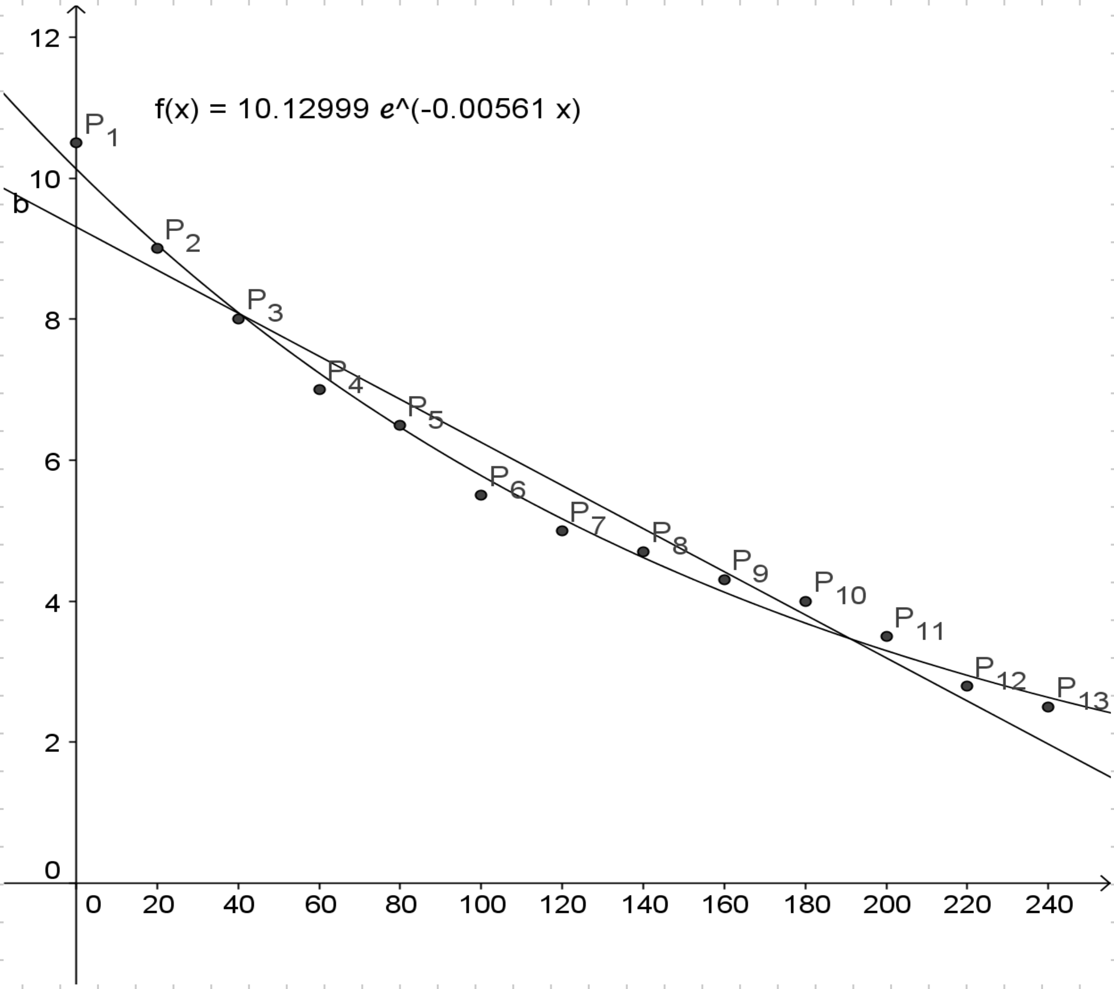
A: In 6,4 Jahren ist der Bestand v. 2000 Elefanten wieder erreicht.

⇒ Bierschaumzerfall

$5 = 10{,}13 \cdot e^{-0{,}00561 \cdot t}$

⋮

$t =$



Kopie Bsp. 5 (Abkühlung Wasser)

$$T(t) = \underline{T_u} + (T_0 - T_u) \cdot e^{-0,05 \cdot t}$$

T_u ... Umgebungstemp.
T_0 ... Ausgangstemp.
e ... Euler'sche Zahl

$T_0 = 22°C$
$T_u = -16°C$

$$T(t) = -16 + (22 - (-16)) \cdot e^{-0,05 \cdot t}$$

$$\boxed{T(t) = -16 + 38 \cdot e^{-0,05 \cdot t}}$$

[$\widehat{=}$ $N(t) = N_0 \cdot e^{\lambda t}$]

$\frac{\ln 2}{\lambda} = \tau$

$$T(t_1) = 0 = -16 + 38 \cdot e^{-0,05 \cdot t_1} \quad | +16$$

$$16 = 38 \cdot e^{-0,05 \cdot t_1} \quad | :38$$

$$\ln \frac{16}{38} = \ln e^{-0,05 \cdot t_1}$$

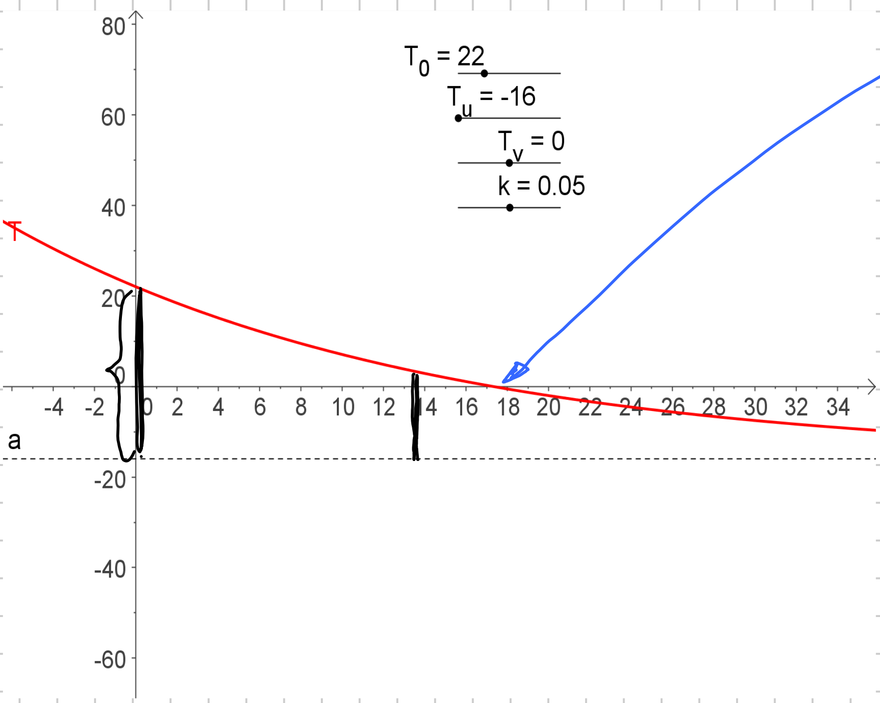
$$\ln \frac{16}{38} = -0,05 \cdot t_1 \cdot \overset{1}{\ln e} \quad | : -0,05$$

$$\frac{\ln \frac{16}{38}}{-0,05} = t_1 = +17,29$$

$$\approx 18 \text{ Minuten}$$

A: Es dauert rund 18 Minuten

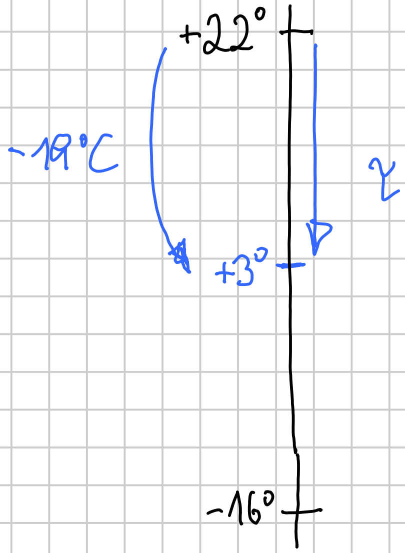
b)



c) $T(\tau) \neq 11°C$ $\qquad 22-(-16) = 38°C$

$19°C$

$$\boxed{T(\tau) = 3°C}$$



$$3 = -16 + 38 \cdot e^{-0{,}05 \cdot \tau} \quad | +16$$

$$19 = 38 \cdot e^{-0{,}05 \cdot \tau} \quad | :38$$

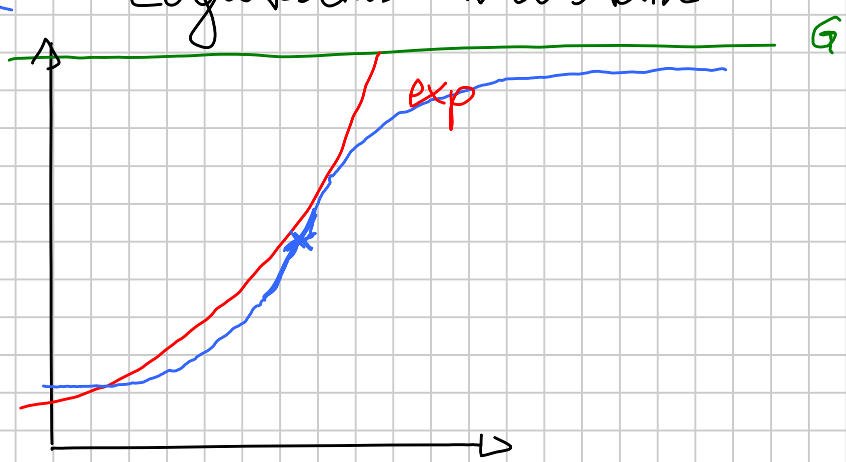
$$\ln \frac{1}{2} = \ln e^{-0{,}05 \cdot \tau}$$

$$\ln \frac{1}{2} = \overset{0}{\ln 1} - \ln 2$$

$0{,}86 \cdot 60s$

$$\frac{\ln \frac{1}{2}}{-0{,}05} = \tau = 13{,}86 \text{ min} \mathrel{\hat{=}} 13 \text{ min } 52 s$$

S 4/12 Logistisches Wachstum



S 4/15 B. 1.15

$$N(t) = \frac{50.000}{1 + 49.999 \cdot e^{k \cdot t}}$$

$$N(3) = 1745 = \frac{50.000}{(1 + 49.999 \cdot e^{k \cdot 3})} \quad \Big| \cdot (\text{ Nenner })$$

$$1745 \cdot (1 + 49.999 \cdot e^{3k}) = 50000 \quad \Big| : 1745 \quad \Big| -1$$

$$\cancel{1+}\ 49.999 \cdot e^{3k} = \frac{50000}{1745} - 1$$

$$\ln e^{3k} = \ln \left(\frac{\frac{50000}{1745} - 1}{49999} \right)$$

$$3k = -7{,}5000 \ldots$$

$$\underline{\underline{k \approx -2{,}5}} \qquad \boxed{STO} \quad \boxed{k}$$

b) $$N(t_1) = 1\cancel{0000} = \frac{5\cancel{0000}}{1 + 49999 \cdot e^{k \cdot t_1}} \quad \Big| \cdot (\qquad)$$

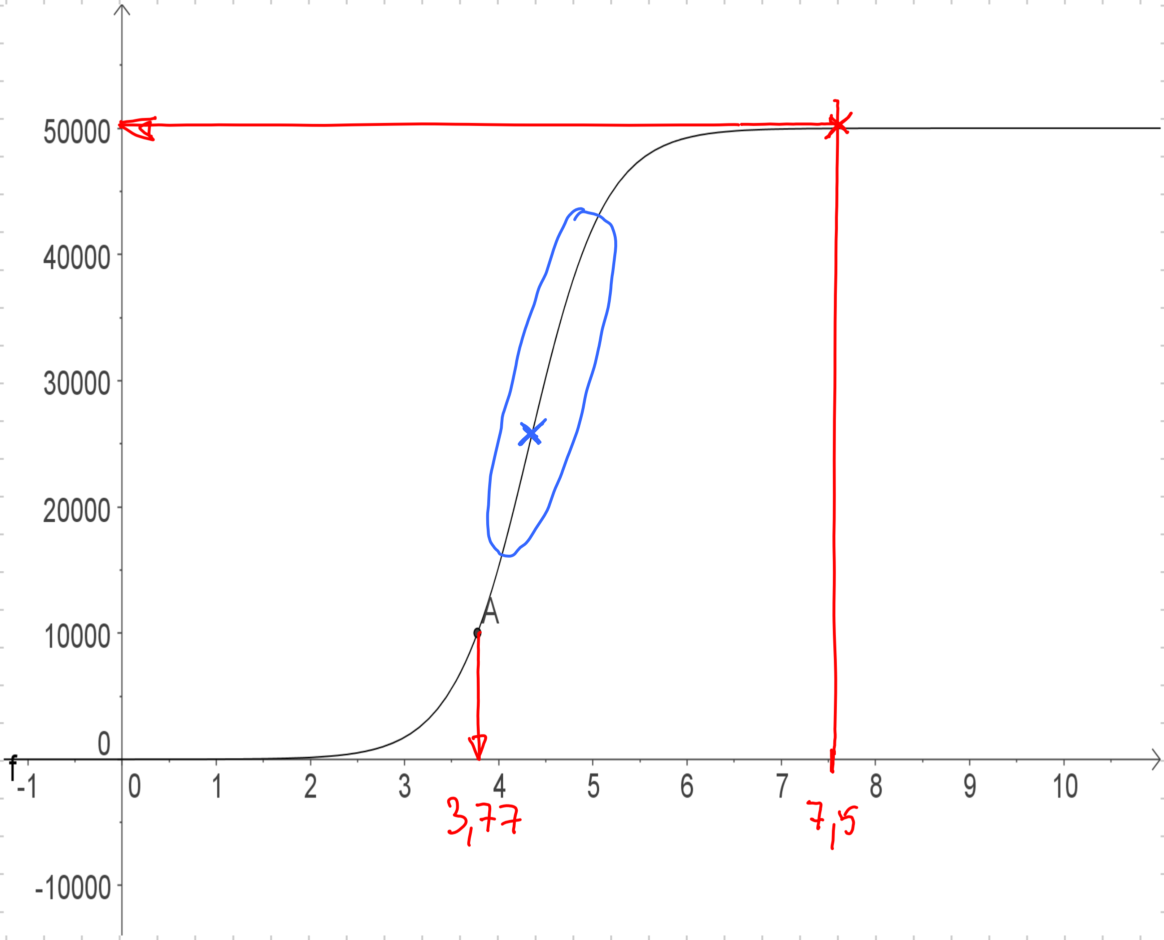
$$\cancel{1+}\ 49.999 \cdot e^{k t_1} = \cancel{5}\ 4$$

$$\ln e^{k t_1} = \ln \frac{4}{49.999}$$

$$t_1 = \frac{\ln \frac{4}{49999}}{k}$$

$$t_1 = 3{,}77$$

A: Nach 3,8 Tagen kennen 10.000 Personen das Gerücht

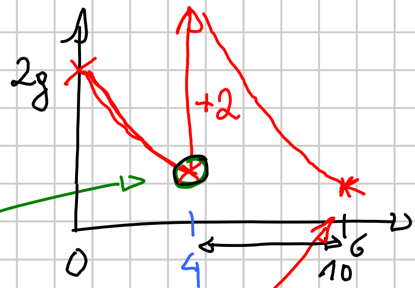


B.1.17

$$\tau = -\frac{\ln 2}{\lambda} \qquad N(t) = N_0 \cdot e^{\lambda \cdot t}$$

$$\lambda = -\frac{\ln 2}{\tau} = -\frac{\ln 2}{3} = -0{,}23104\ldots \qquad \boxed{STO} \quad \boxed{L}$$

8 Uhr $\Rightarrow t_0$ $\qquad N_0 = 2g$



$$N(t) = 2 \cdot e^{\lambda \cdot t}$$

$$N(4) = 2 \cdot e^{\lambda \cdot 4}$$

$$N(4) = 0{,}7937\ldots$$

$\downarrow +2$

$$\Rightarrow \overline{N_0} = 2{,}7937$$

$$N(6) = \overline{N_0} \cdot e^{\lambda \cdot 6}$$

18 Uhr $\quad N(6) = 0{,}6984\ldots g$

A: Um 18 Uhr sind 0,6984 g ACSS im Körper

20Uhr: $N(8) = \overline{\overline{N_0}} \cdot e^{\lambda \cdot 8}$

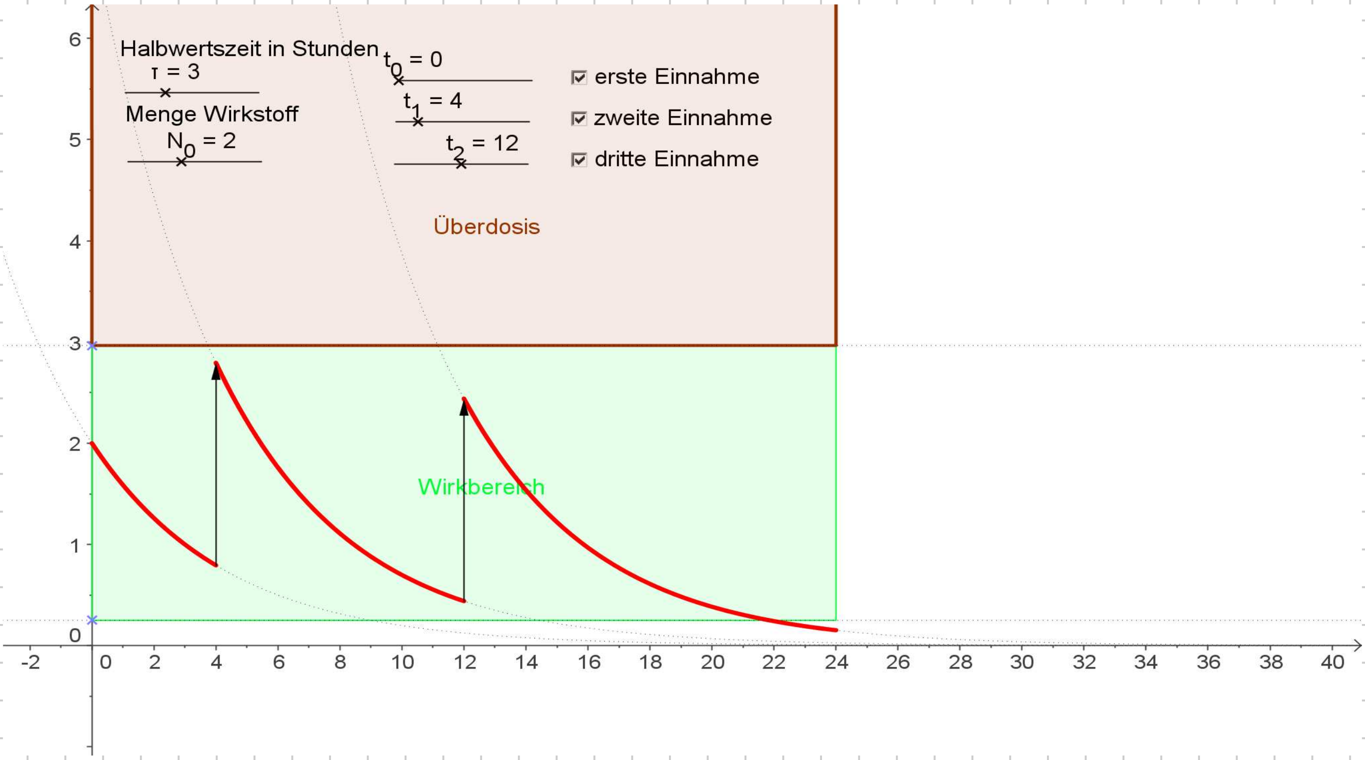
$N(8) = 0{,}43998$ $+2$

$\overline{\overline{N_0}} = 2{,}43998\,g$

24Uhr: $N(4) = \overline{\overline{N_0}} \cdot e^{\lambda \cdot 4}$

$N(4) = 0{,}9682\,g$

A: Um 24 Uhr sind 0,968 g ACSS im Körper



Kopie B.10

$B_0 = 380$ $\qquad$ $\underline{B(45) = 210}$

$$B(t) = 1 + \left(B_0 + \sqrt{B_0}\right) \cdot e^{\lambda t}$$

$$210 = 1 + \left(380 + \sqrt{380}\right) \cdot e^{\lambda \cdot 45}$$

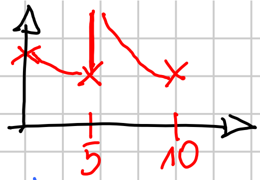
$$209 = \left(380 + \sqrt{380}\right) \cdot e^{\lambda \cdot 45} \quad \Big| : \left(380 + \sqrt{380}\right)$$

$$\ln 0{,}5231\ldots = \ln e^{45 \cdot \lambda}$$

$$\frac{\ln 0{,}5231}{45} = \lambda = -0{,}0143969\ldots$$

$$\underline{\underline{-0{,}014397}}$$

b) $B(5) = 1 + \left(380 + \sqrt{380}\right) \cdot e^{\lambda \cdot 5}$



$B(5) = 372{,}74$ $\quad +290$

1+(380+√(380))*e^(λ*5)

$\overline{B_0} = 662{,}74$

$\overline{B}(5) = 1 + \left(\overline{B_0} + \sqrt{\overline{B_0}}\right) \cdot e^{\lambda \cdot 5}$

$\overline{B(5)} = 641{,}6 \approx 642$ Bläschen

c) $B(10) = 1 + \left(380 + \sqrt{380}\right) \cdot e^{\lambda \cdot 10}$

$B(10) = 346{,}92$ $\quad + 290$

$\overline{B(10)} = 636{,}9 \approx 637$ Bläschen

d) $318 = 1 + \left(380 + \sqrt{380}\right) \cdot e^{\lambda \cdot t_1}$

$317 = \left(380 + \sqrt{380}\right) \cdot e^{\lambda} \quad \Big| : (\quad)$

$\ln 0{,}7935 = \ln e^{\lambda \cdot t_1}$

$\ln 0{,}7935 = \lambda \cdot t_1$

$\frac{\ln 0{,}7935}{\lambda} = t_1$

$16{,}06 = t_1$

A: Er muss 16 Minuten warten.

11)

$$N(t) = \frac{N_0 \cdot G}{N_0 + (G - N_0) \cdot e^{k \cdot t}}$$

$G = 10.000$
$N_0 = 1130$
$N(3) = 2000$

$$2\cancel{0}\cancel{0}\cancel{0} = \frac{1130 \cdot 10.\cancel{0}\cancel{0}\cancel{0}}{1130 + 8870 \cdot e^{k \cdot 3}}$$

$$2(1130 + 8870 \cdot e^{3k}) = 11300 \quad | :2$$

$$1130 + 8870 \cdot e^{3k} = 5650 \quad | -1130$$

$$8870 \cdot e^{3k} = 4520$$

$$\ln e^{3k} = \ln \frac{4520}{8870}$$

$$3k = \ln \frac{452\cancel{0}}{887\cancel{0}} \quad | :3$$

$$k = -0{,}22472$$

b) $$N(5) = \frac{1130 \cdot 10.000}{1130 + 8870 \cdot e^{k \cdot 5}} = 2815{,}36$$

A: Es sind 2815 PCs nach 5 Tage infiziert

c) $$70\% \text{ v. } 10.000 = 7\cancel{0}\cancel{0}\cancel{0} = \frac{1130 \cdot 10.\cancel{0}\cancel{0}\cancel{0}}{1130 + 8870 \cdot e^{k \cdot t}}$$

$$7 \cdot (1130 + 8870 \cdot e^{k \cdot t}) = 11300 \quad | :7 \,| -1130$$

$$8870 \cdot e^{k \cdot t} = 484{,}28$$

$$k \cdot t = \ln 0{,}05459$$

$$t = 12{,}939$$

→ A: Nach ca. 13 Tagen

12) $F_0 = 1570$ db

a) $F(20) = \frac{1}{2} N_0 \Rightarrow \lambda = -\frac{\ln 2}{20} = -0{,}0346$

$$N(m) = 1570 \cdot e^{-0{,}0346 \cdot m}$$

b) $300 = 1570 \cdot e^{\lambda \cdot m}$

$$\ln \frac{30\cancel{0}}{157\cancel{0}} = \ln e^{\lambda \cdot m}$$

$$\ln \frac{30}{157} = \lambda m \quad | : \lambda$$

$$47{,}75\,m = m$$

A: Nein geht sich nicht aus!

(vgl. mit 55m)

c) $\Rightarrow$ siehe b

d) $0{,}3 \cdot N_0 \stackrel{?}{=} N_0 \cdot e^{\lambda \cdot 25}$

$0{,}3 \cdot N_0 \stackrel{?}{=} N_0 \cdot 0{,}42$

$30\% \neq 42\%$ $\Rightarrow$ Gerät defekt