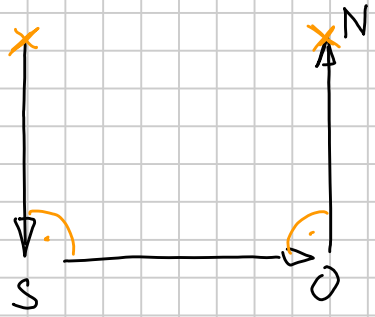
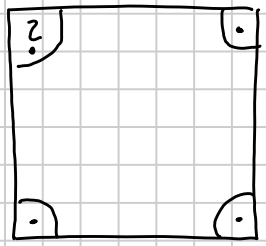
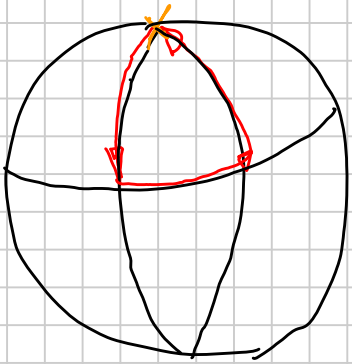
Denksport

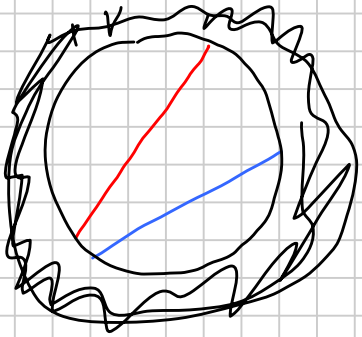
Trigonometrie

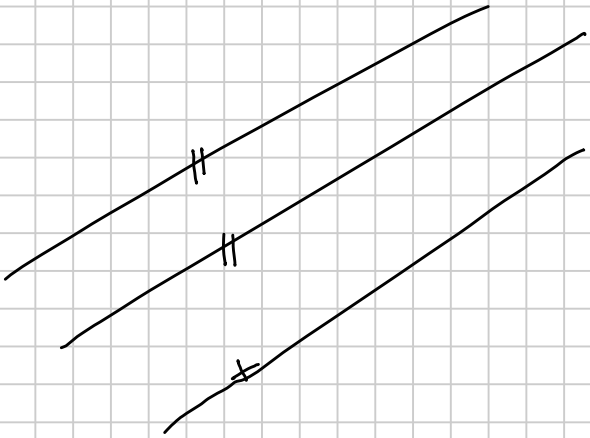


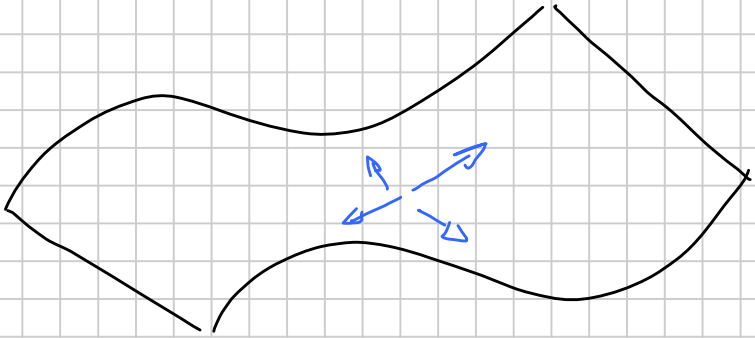


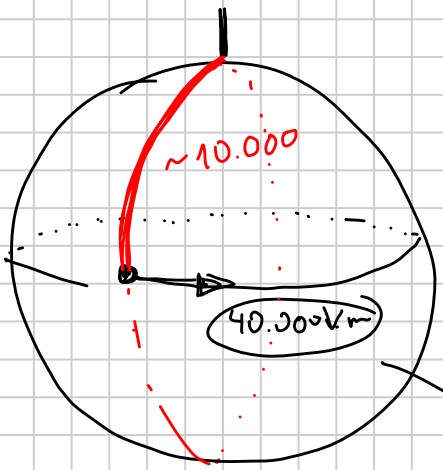
Nicht - Euklidische Geometrie



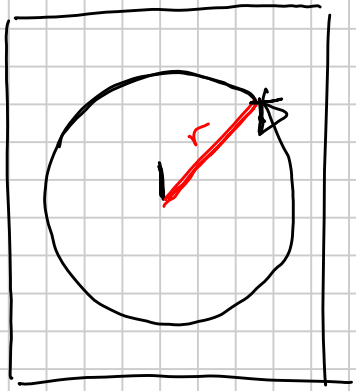








3,1415



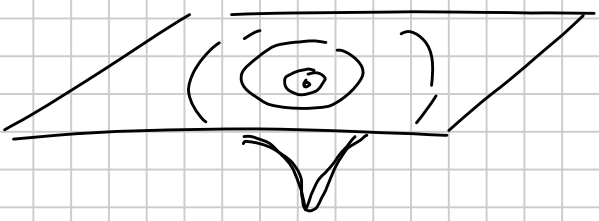
$u = 2r \cdot \pi$

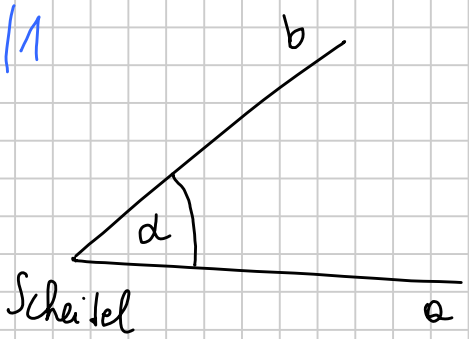
$u = 2 \cdot 1 \cdot \pi$

$u = 6{,}28m$

$2 \cdot 10.000 \cdot \pi$

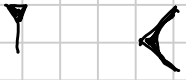
$20.000\,\pi \approx 60.000\,km$





Grad °

360°



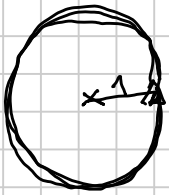
60

90° = 100 Neugrad (gon) g

100g

⇒ Vollwinkel

360° = 400g = 2π Radiant



$u = 2r\pi$

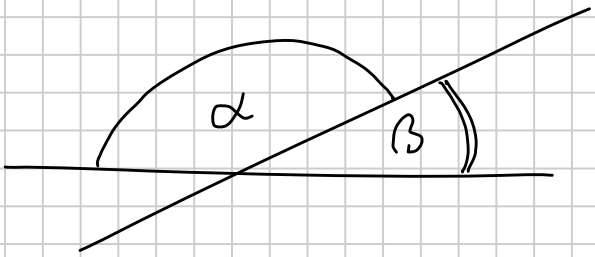
$\alpha, \beta, \gamma, \delta, \varepsilon, \varphi, \dots \psi, \mu, \sigma, \varrho, \lambda, \xi$

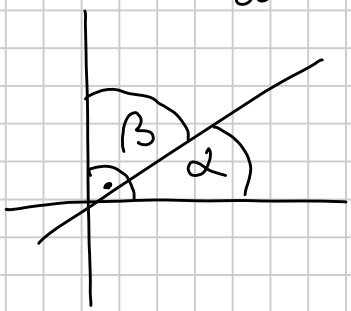
phi psi mü sigma rho

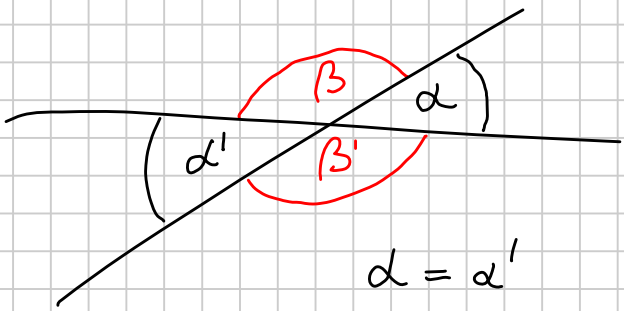
45° 17' 38,4"

$\frac{1}{60}° = 1'$

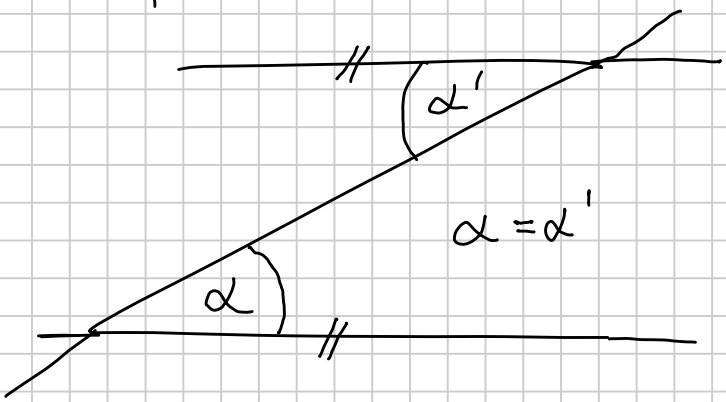
$\frac{1}{60}' = 1''$



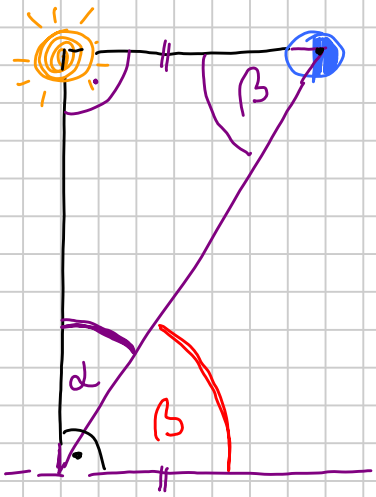




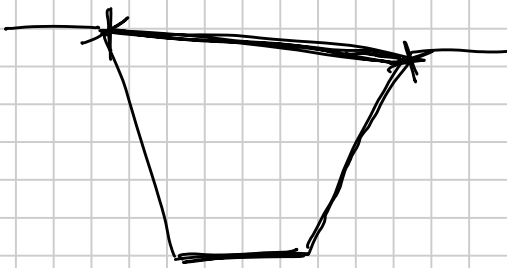
$\alpha = \alpha'$



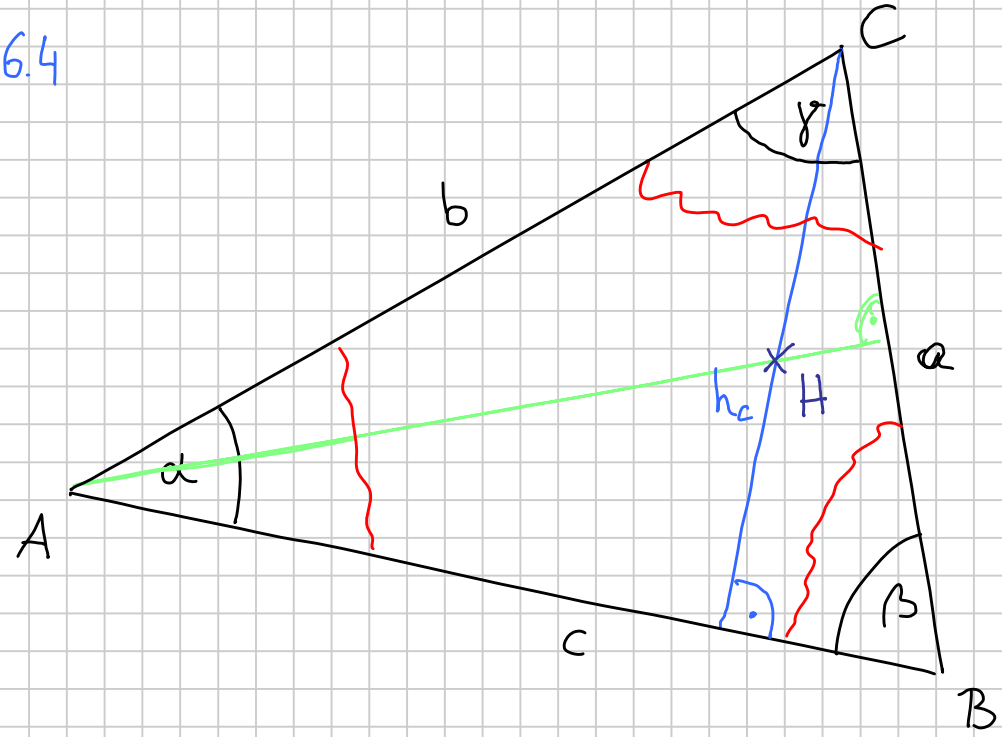
$\alpha = \alpha'$



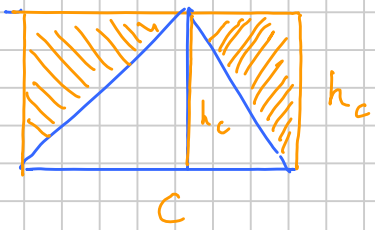
$\beta = 90 - \alpha$

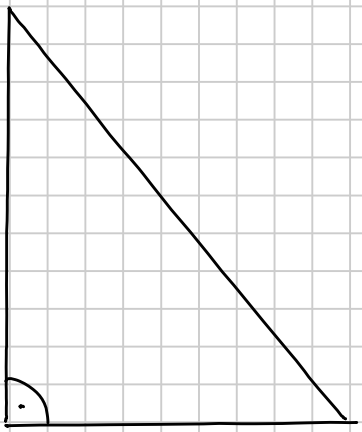


S. 6.4



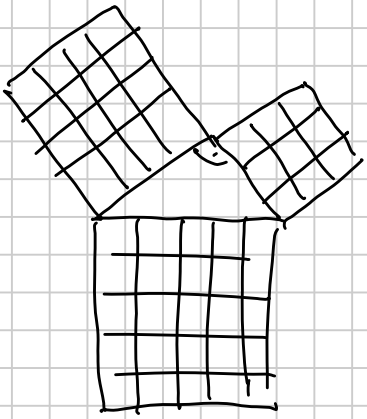
$$A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$$





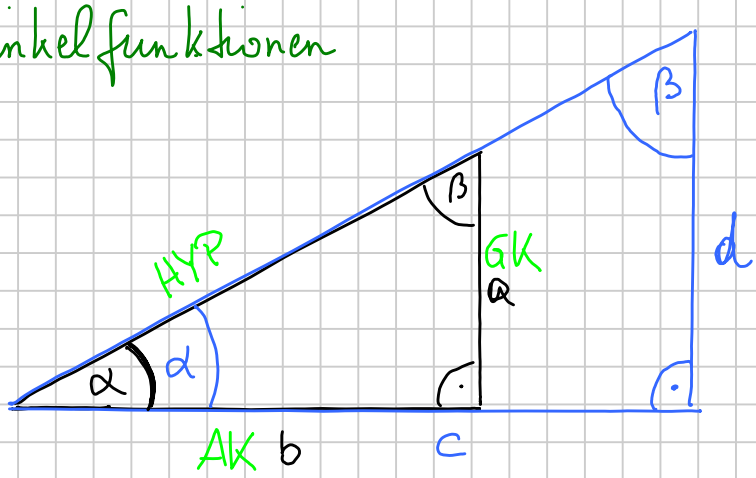
$a^2 + b^2 = c^2$

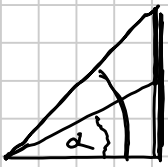
$[E = m \cdot c^2]$



3, 4, 5

Winkelfunktionen





$$a : b = d : c$$

$$\frac{a}{b} = \frac{d}{c}$$

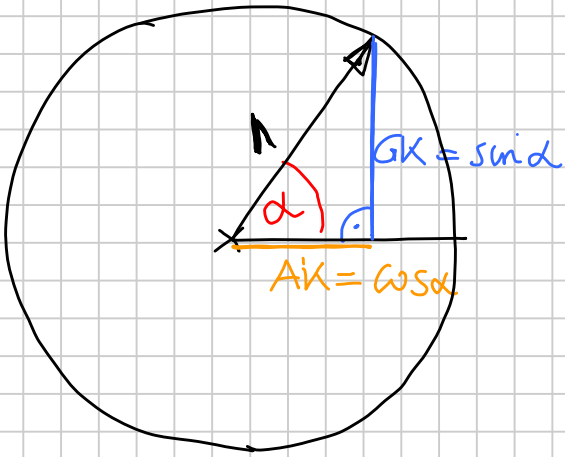
$$\tan\alpha = \frac{GK}{AK}$$

$$\sin\alpha = \frac{GK}{HYP}$$

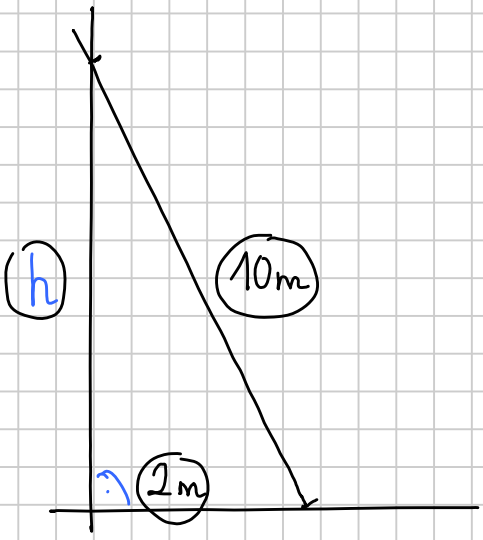
$$\cos\alpha = \frac{AK}{HYP}$$

Sinus (lat. Welle)

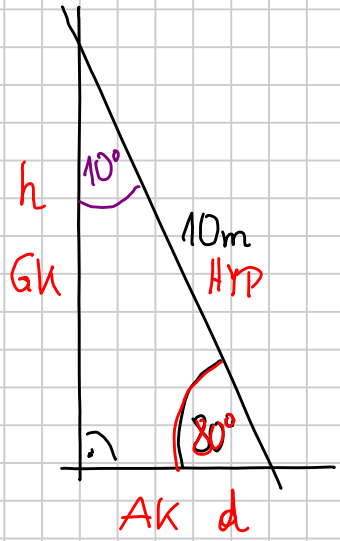
Einheitskreis



$$\frac{GK}{1} = \sin\alpha = GK$$



$$h^2 + 2^2 = 10^2$$
$$h^2 = 10^2 - 2^2$$
$$h^2 = 96$$
$$h = \sqrt{96} = 9{,}797\,m$$



h: $\sin\alpha = \frac{GK}{HYP}$

$\sin 80° = \frac{h}{10} \quad | \cdot 10$

$10 \cdot \sin 80° = h$

$9{,}848\,m = h$

d: $\cos\alpha = \frac{AK}{HYP}$

$\cos 80° = \frac{d}{10} \quad | \cdot 10$

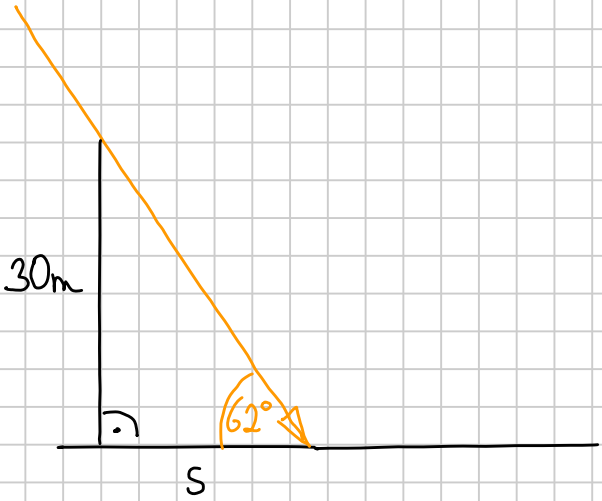
$10 \cdot \cos 80° = d$

$1{,}736\,m = d$

Probe: $\frac{GK}{AK} = \frac{9{,}848}{1{,}736} = 5{,}672 \overset{?}{=} \tan 80°$

$\tan 80° = 5{,}671$ ✓

S 6/7



$\tan\alpha = \frac{GK}{AK}$

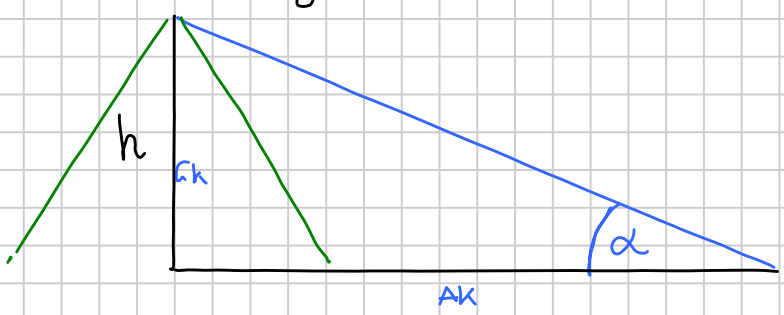
$\tan 62° = \frac{30}{s} \quad | \cdot s$

$s \cdot \tan 62° = 30 \quad | : \tan 62°$

$s = \frac{30}{\tan 62°}$

$s = 15{,}95\,m$

Bsp. Pyramidenhöhe 146m aus 2km Entfernung



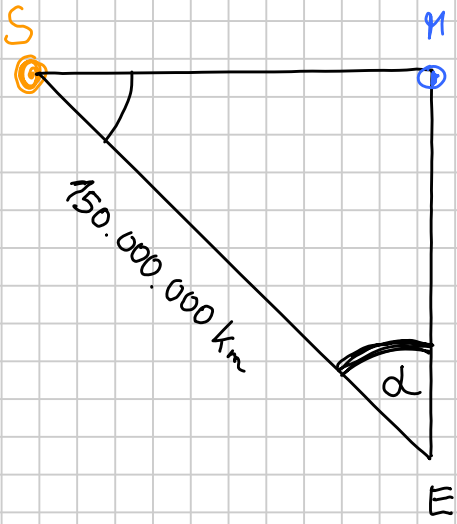
$\tan\alpha = \frac{GK}{AK}$

$\tan\alpha = \frac{146}{2000} = 0{,}073$

$\alpha = \arctan 0{,}073$

$\alpha = 4{,}17°$

Bsp. Halbmond



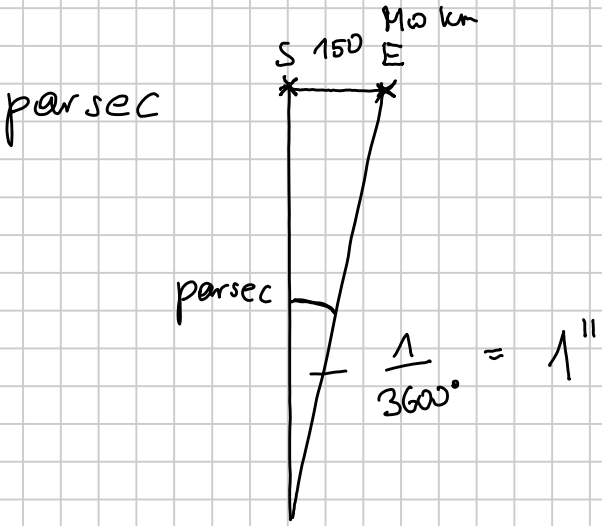
384.400km

$\cos\alpha = \frac{AK}{HYP}$

$\cos\alpha = \frac{384.400}{150.000.000}$

$\alpha = \arccos 0{,}00\ldots$

$\alpha = 89{,}853°$



$\tan\frac{1}{3600} = \frac{150\cdot 10^6}{parsec}$

$parsec = \frac{150\cdot 10^6}{\tan\frac{1}{3600}}$

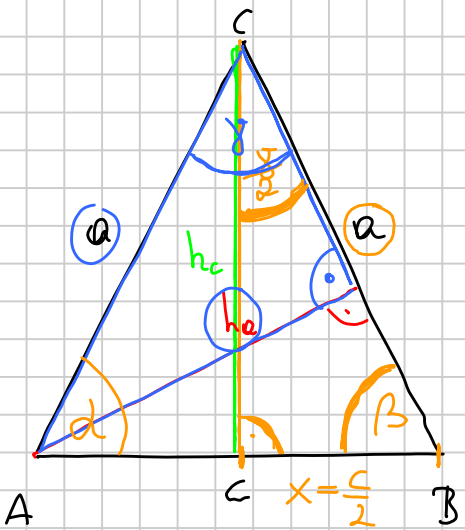
$parsec = 3{,}09\cdot 10^{13}$

~ 30000000000000 km

S. 6/8

$a = 405$ mm

$A = 78732$ mm²



$A = \frac{a\cdot h_a}{2} \mid \cdot 2$

$2A = a\cdot h_a \mid : a$

$\frac{2A}{a} = h_a$

$h_a = 388{,}8$ mm

$\sin\gamma = \frac{h_a}{a}$

$\Rightarrow \gamma = \arcsin\frac{h_a}{a} = 73{,}739$ C

$\sin\left(\frac{\gamma}{2}\right) = \frac{x}{a}$

$\Rightarrow a\cdot\sin\frac{\gamma}{2} = x \Rightarrow c = 2x$

$243\,mm = x \Rightarrow c = 486\,mm$

$\beta = 90 - \frac{\gamma}{2} = 53{,}13°$

$\beta = \frac{180 - \gamma}{2} = 53{,}13°$

$\tan\beta = \frac{h_c}{x} \Rightarrow x \cdot \tan\beta = h_c$

$h_c = 324\,mm$

B. 3.7e

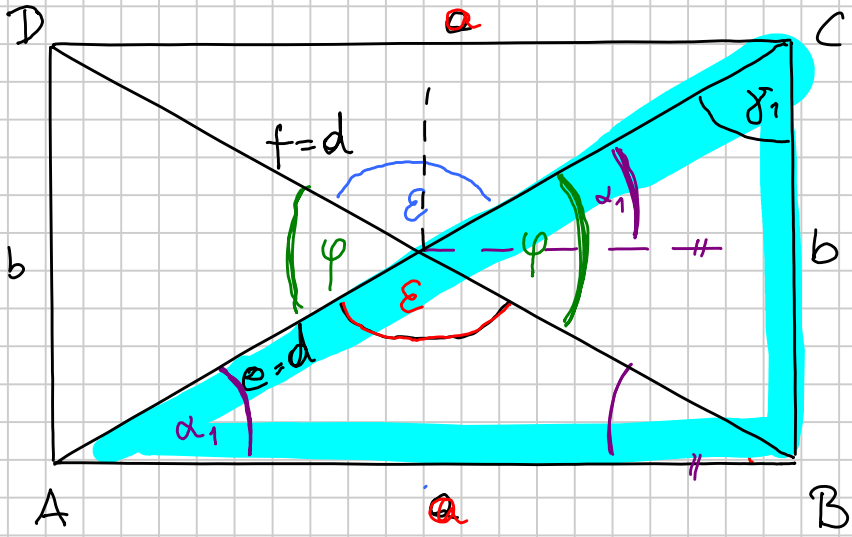
$a = 12$

$\varepsilon = 134{,}76°$

$\varphi = 180 - \varepsilon$

$\alpha_1 = \frac{\varphi}{2}$ od. $\frac{180-\varepsilon}{2}$

$\gamma_1 = 90 - \alpha_1 = \frac{\varepsilon}{2}$



$\tan\alpha_1 = \frac{b}{a} \Rightarrow b = a \cdot \tan\alpha_1$

$A = a \cdot b$

$d^2 = a^2 + b^2 \Rightarrow d = \sqrt{a^2 + b^2}$

od. $\cos\alpha_1 = \frac{a}{d} \Rightarrow d = \frac{a}{\cos\alpha_1}$

B. 3.8. d

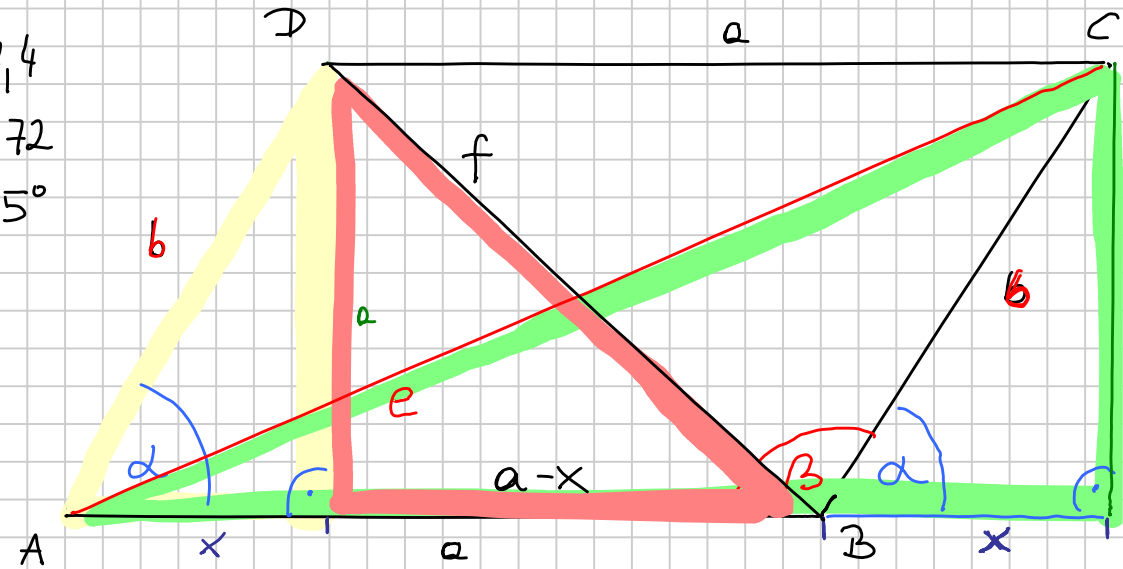
$b = 27{,}4$

$e = 76{,}72$

$\beta = 105°$

$\alpha = 180 - \beta$

$A = a \cdot h_a$



$$\sin\alpha = \frac{h_a}{b} \quad \Rightarrow \quad h_a = b \cdot \sin\alpha$$

$$\cos\alpha = \frac{x}{b} \quad \Rightarrow \quad x = b \cdot \cos\alpha$$

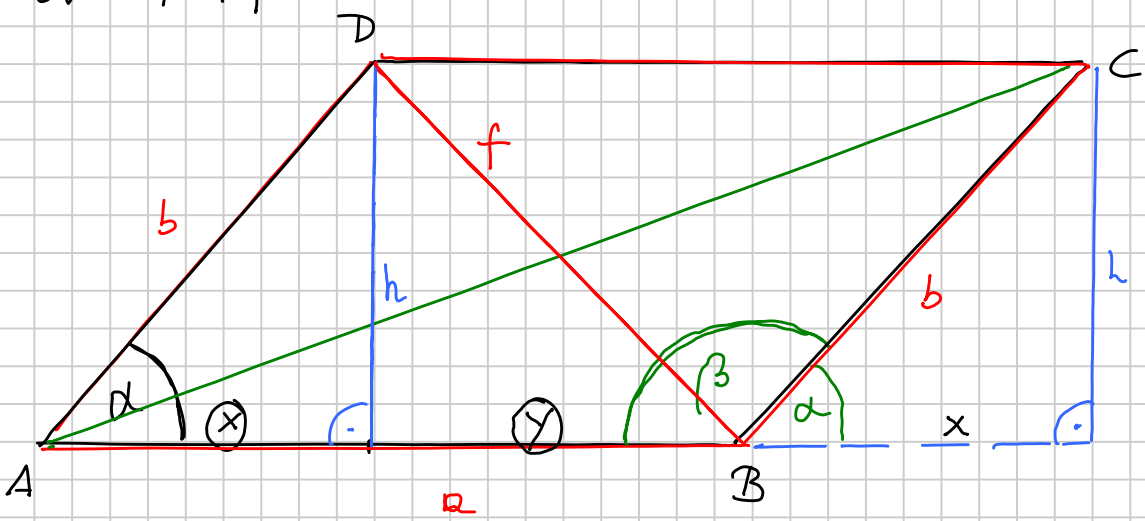
$$(a+x)^2 + h_a^2 = e^2$$

$$a+x = \sqrt{e^2 - h_a^2}$$

$$a = \sqrt{e^2 - h_a^2} - x$$

$$f^2 = (a-x)^2 + h_a^2 \quad \Rightarrow \quad f = \sqrt{(a-x)^2 + h_a^2}$$

B. 3.8b. Geg.: a, b, f



I: $b^2 = h^2 + x^2$ $\qquad y = a - x$ $\qquad x + y = a$

II: $f^2 = y^2 + h^2$

I: $h^2 = b^2 - x^2$ $\rightarrow$ $h = \sqrt{b^2 - x^2}$

II: $h^2 = f^2 - y^2 = f^2 - (a-x)^2$

$$h^2 = h^2$$

$$b^2 - x^2 = f^2 - (a-x)^2$$

$$b^2 - x^2 = f^2 - (a^2 - 2ax + x^2)$$

$$b^2 - x^2 = f^2 - a^2 + 2ax - x^2$$

$$b^2 - f^2 + a^2 = 2ax \quad | : 2a$$

$$\frac{b^2 - f^2 + a^2}{2a} = x$$

$$\cos\alpha = \frac{x}{b} \Rightarrow \alpha = \arccos\frac{x}{b}$$

$\cos^{-1}$

$$\beta = 180 - \alpha$$

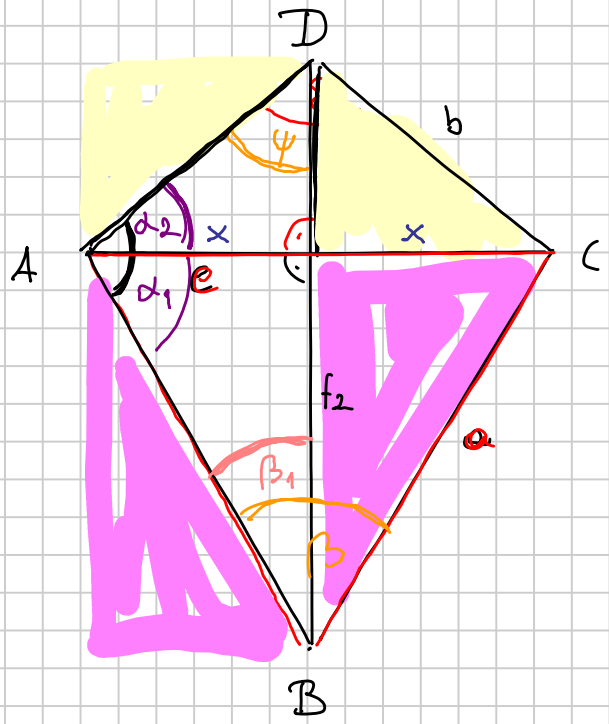
$$(a+x)^2 + h^2 = e^2 \Rightarrow e = \sqrt{(a+x)^2 + h^2}$$

$$A = a \cdot h$$

Bsp. Deltoid

Geg.: e, δ, a

Ges. b, α, β, f, A



$$x = \frac{e}{2} \qquad \psi = \frac{\delta}{2}$$

$$\sin\psi = \frac{x}{b} \Rightarrow b = \frac{x}{\sin\psi}$$

$$\alpha_2 = 90 - \psi$$

$$\cos\alpha_1 = \frac{x}{a} \Rightarrow \alpha_1 = \arccos\frac{x}{a}$$

$\cos^{-1}$

$$\alpha = \alpha_1 + \alpha_2$$

$$\beta_1 = 90 - \alpha_1 \qquad \beta = 2 \cdot \beta_1$$

$$b^2 - x^2 = f_1^2 \Rightarrow f_1 = \sqrt{b^2 - x^2}$$

$$a^2 - x^2 = f_2^2 \Rightarrow f_2 = \sqrt{a^2 - x^2}$$

$$f_1 + f_2 = f$$

$$A = \frac{e \cdot f}{2}$$