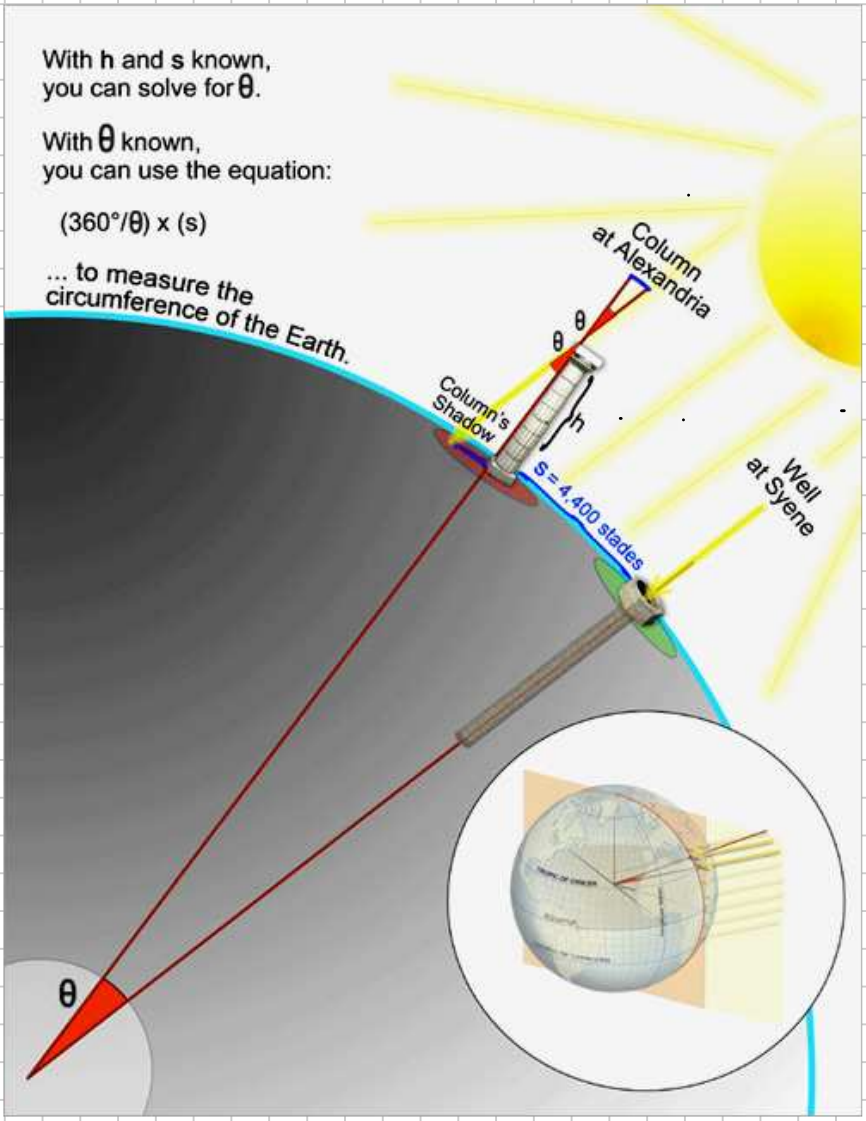
Trigonometrie

BRP Mathematik - Mag. Kurt Söser

Denksport-Wiederholung

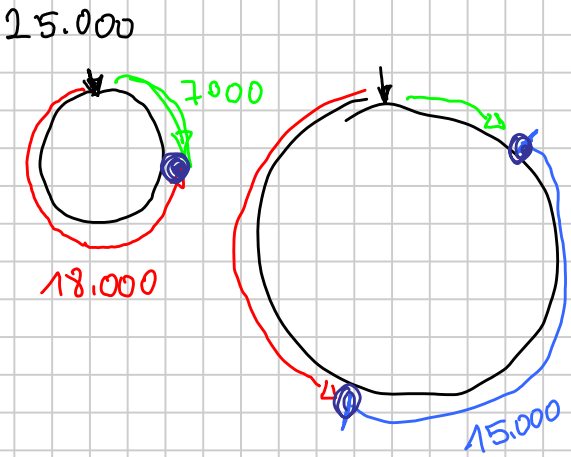
Umfang der Erde ? Eratosthenes



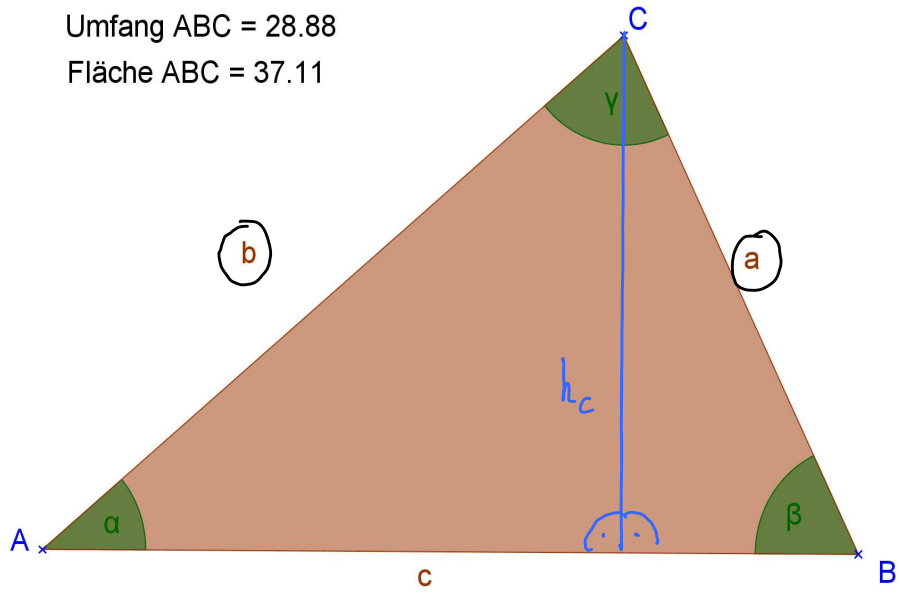
50 ↦ 7,2° ≙ 4.400

360° ≙ 220.000

≙ 40.040 km



Allgemeines Dreieck S 6/13



Sinus-Satz

$$\sin\alpha = \frac{h_c}{b} \Rightarrow h_c = b \cdot \sin\alpha$$

$$\sin\beta = \frac{h_c}{a} \Rightarrow h_c = a \cdot \sin\beta$$

$$h_c = h_c \quad | : a$$

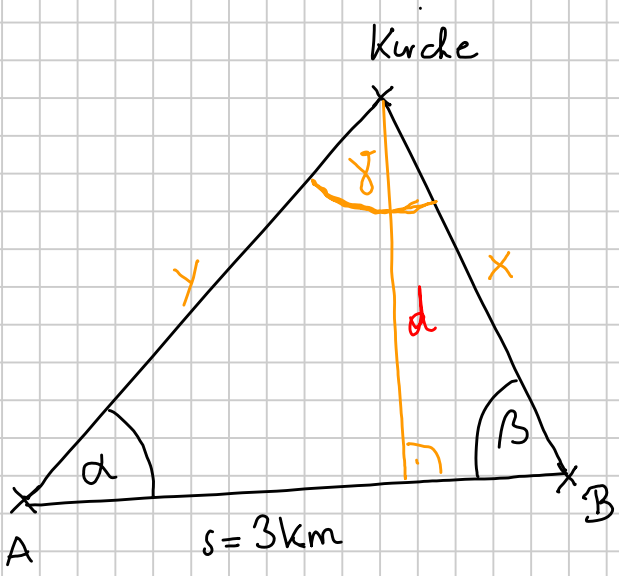
$$b \cdot \sin\alpha = a \cdot \sin\beta \quad | : \sin\alpha \quad | : \sin\beta$$

$$\boxed{\frac{b}{\sin\beta} = \frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}}$$

SSW
SWW

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Bsp.



$\alpha = 53°$

$\beta = 65°$

$s = 3\,km$

$\gamma = 180 - \alpha - \beta$

$$\frac{x}{\sin\alpha} = \frac{s}{\sin\gamma} \Rightarrow x = \frac{s \cdot \sin\alpha}{\sin\gamma}$$

$$x = \frac{3 \cdot \sin 53}{\sin 62°} = 2{,}713 \text{ km}$$

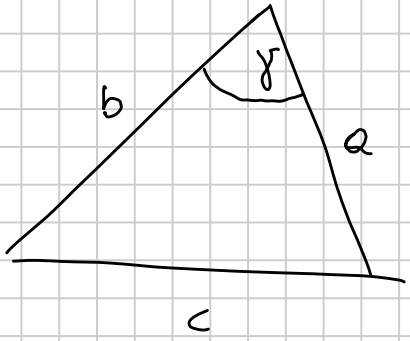
$$\frac{y}{\sin\beta} = \frac{s}{\sin\gamma} \Rightarrow y = \frac{s \cdot \sin\beta}{\sin\gamma}$$

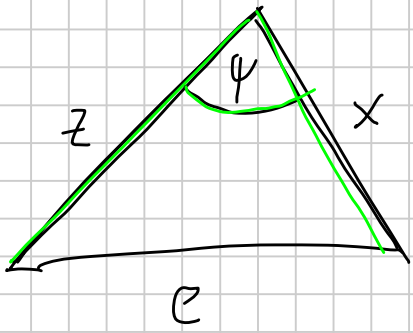
$$\sin\beta = \frac{d}{x} \Rightarrow d = x \cdot \sin\beta$$

Cos-Satz S. 6.17

SSS
SWS

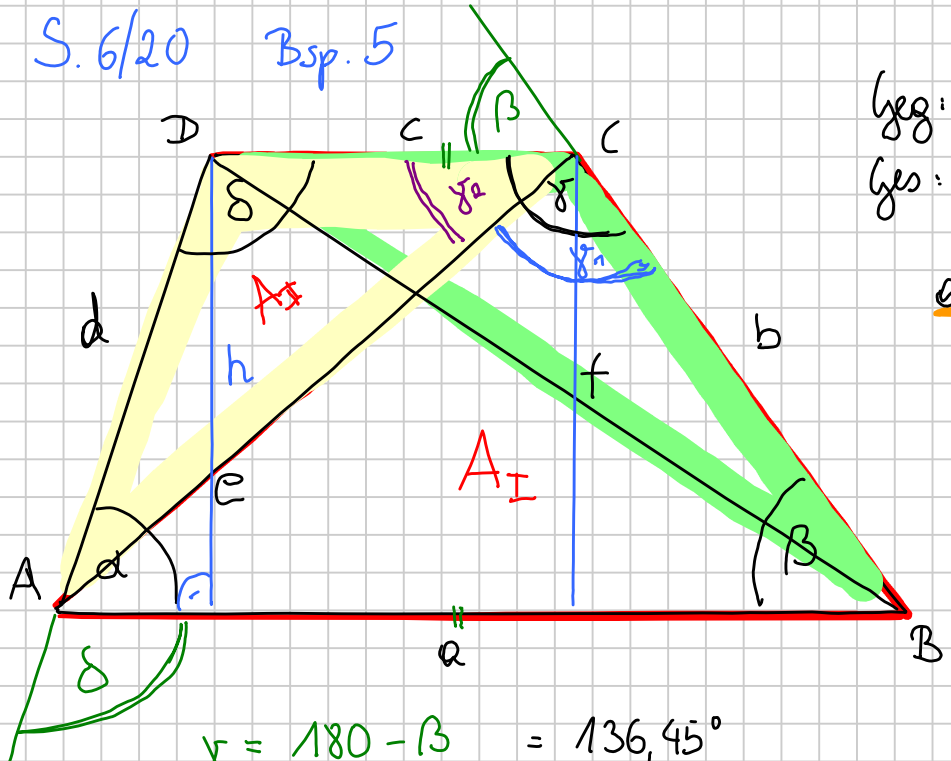
$$c^2 = a^2 + b^2 - 2ab \cdot \cos\gamma$$





$$e^2 = x^2 + z^2 - 2 \cdot x \cdot z \cdot \cos\psi$$

S. 6/20 Bsp. 5



gegeben: a, b, c, e

gesucht: d, α, β, γ, δ, f, A

$$a^2 + b^2 - 2ab \cdot \cos\beta = e^2$$

$$-2ab \cdot \cos\beta = e^2 - a^2 - b^2$$

$$\cos\beta = \frac{e^2 - a^2 - b^2}{-2ab}$$

$$\cos\beta = 0{,}724719\ldots$$

$$\arccos 0{,}7247 = \beta$$

$$43{,}55° = \beta$$

$$\gamma = 180 - \beta = 136{,}45°$$

$$\gamma_2 = \gamma - \gamma_1 = 77{,}98°$$

$$\frac{\sin\gamma_1}{a} = \frac{\sin\beta}{e} \Rightarrow \sin\gamma_1 = \frac{a\cdot\sin\beta}{e}$$

$$\gamma_1 = \arcsin\frac{a\cdot\sin\beta}{e} = 58{,}47$$

- $c^2 + e^2 - 2c\cdot e\cdot\cos\gamma_2 = d^2 \Rightarrow d = \sqrt{\quad}$

$$\sqrt{c^2 + e^2 - 2c\cdot e\cdot\cos\gamma_2} = \underline{\underline{d = 76{,}67\,cm}}$$

- $c^2 + b^2 - 2bc\cdot\cos\gamma = f^2 \Rightarrow f = \sqrt{\ldots} = 52{,}01\,cm$

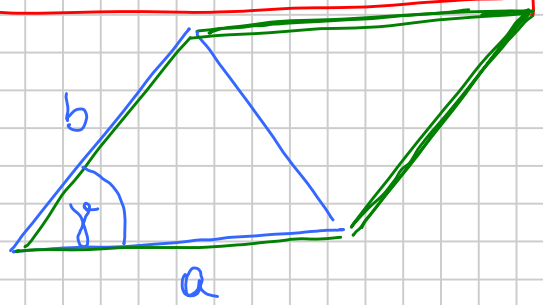
- $\frac{\sin\delta}{e} = \frac{\sin\gamma_2}{d} \Rightarrow \sin\delta = \frac{e\cdot\sin\gamma_2}{d}$

$$\delta = \arcsin\frac{e\cdot\sin\gamma_2}{d}$$

$$\underline{\underline{\delta = 81{,}87°}}$$

- $\alpha = 180° - \delta$
- $A = \frac{a+c}{2}\cdot h$

$$A_\Delta = \frac{a\cdot b\cdot\sin\gamma}{(2)}$$



$$A_I = \frac{a\cdot b\cdot\sin\beta}{2} \qquad A_{II} = \frac{c\cdot d\cdot\sin\delta}{2}$$

$$A_G = A_I + A_{II}$$

5.1.

✓ $\overline{AB} = a$

✓ $\overline{BC} = b$

✓ $\overline{CD} = c$

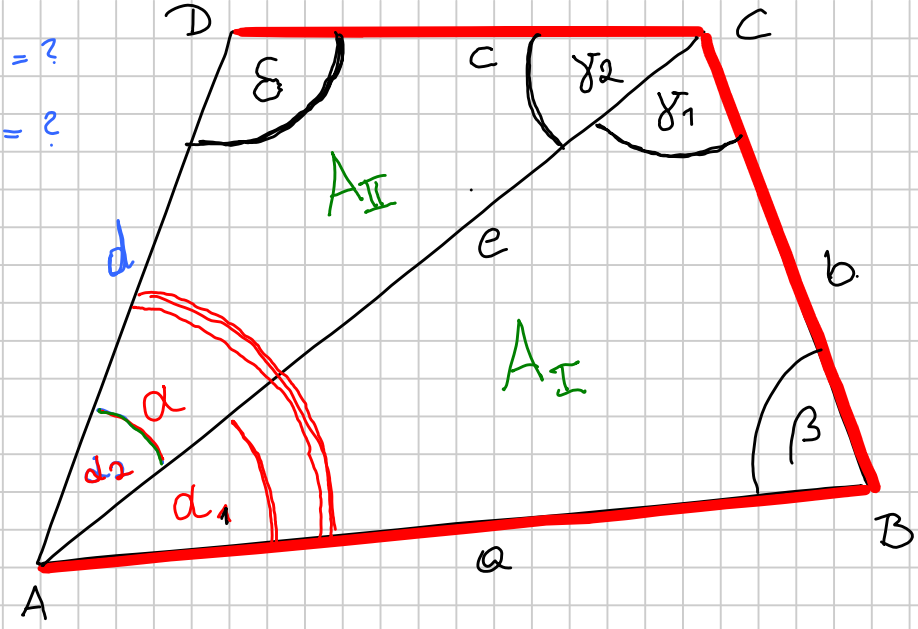
$(\overline{AC} = e)$

$AD = ?$

$A = ?$

$\alpha_1 = 49{,}2°$

$\alpha = 106{,}4°$



$\alpha_2 = \alpha - \alpha_1$

SSW

$$\frac{e}{\sin\beta} = \frac{b}{\sin\alpha_1} \quad \Big| \quad \frac{\sin\gamma_1}{a} = \frac{\sin\alpha_1}{b}$$

$$\boxed{e} = \frac{b \cdot \sin\beta}{\sin\alpha_1} \qquad \sin\gamma_1 = \frac{a \cdot \sin\alpha_1}{b}$$

$$\boxed{\gamma_1} = \arcsin\frac{a \cdot \sin\alpha_1}{b}$$

$$\frac{\sin\delta}{e} = \frac{\sin\alpha_2}{c}$$

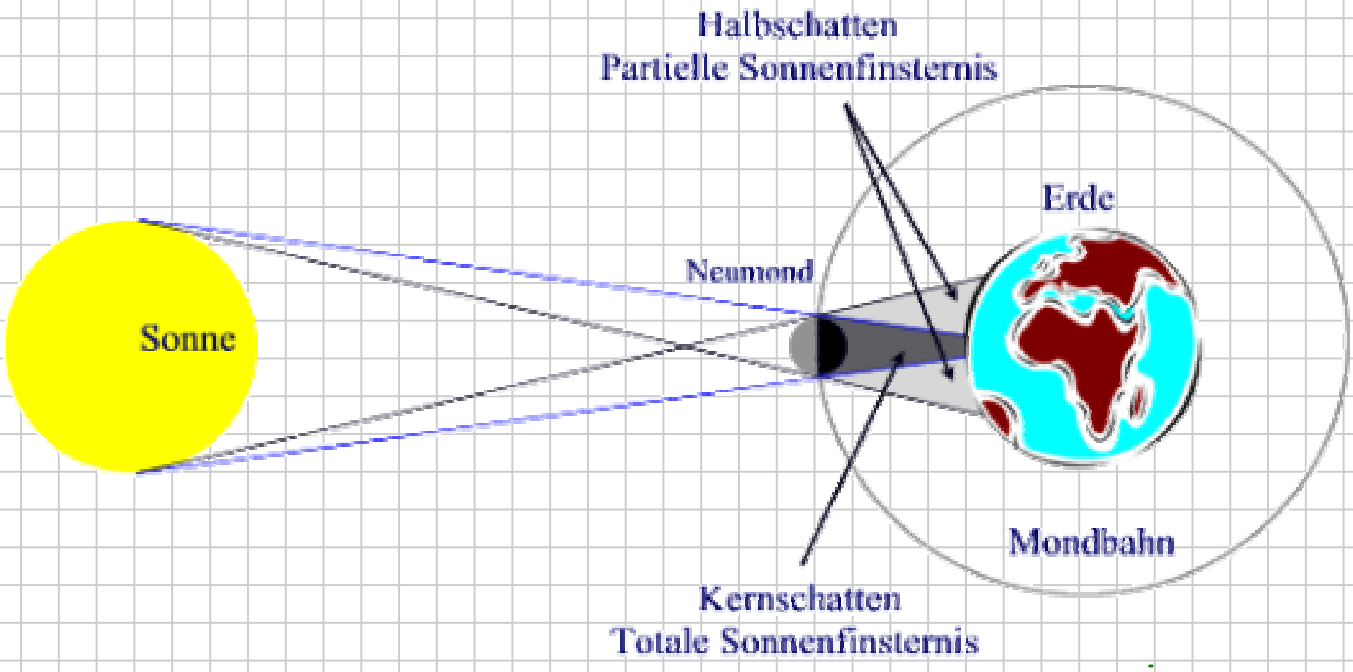
$$\boxed{\beta} = 180 - \gamma_1 - \alpha_1$$

$$\sin\delta = \frac{e \cdot \sin\alpha_2}{c} \quad \Rightarrow \quad \boxed{\delta} = \arcsin\frac{e \cdot \sin\alpha_2}{c}$$

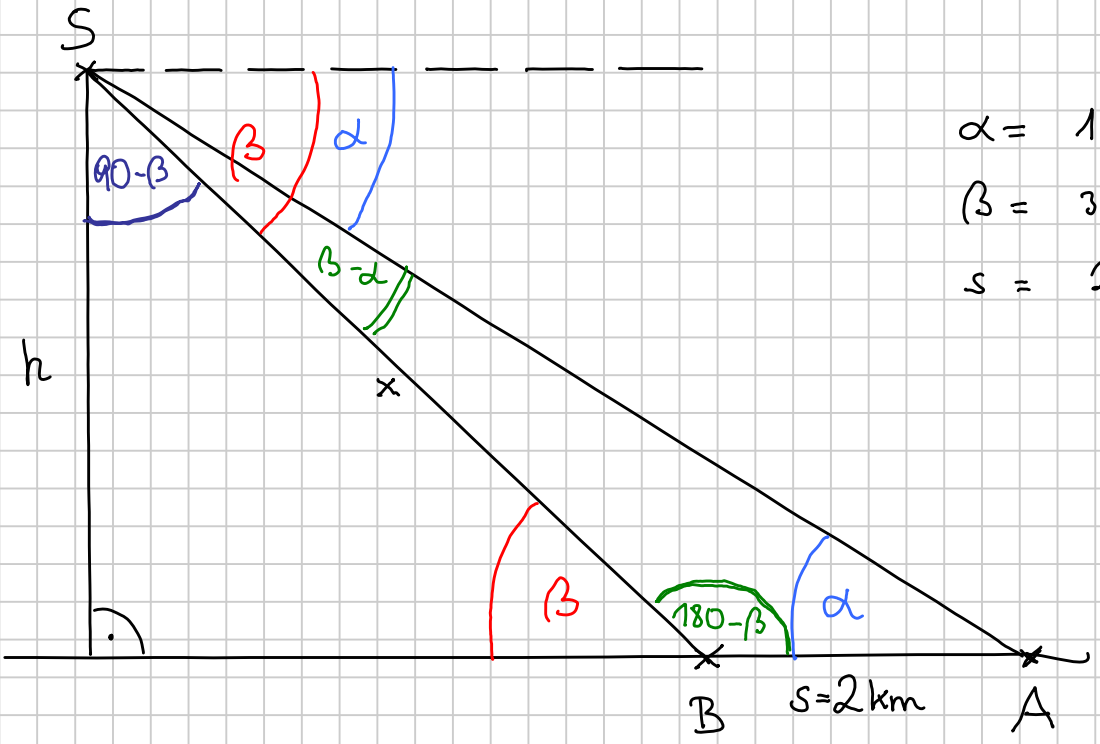
$$\boxed{\gamma_2} = 180 - \delta - \alpha_2$$

$$\frac{d}{\sin\gamma_2} = \frac{c}{\sin\alpha_2} \quad \Rightarrow \quad \boxed{d} = \frac{c \cdot \sin\gamma_2}{\sin\alpha_2}$$

$$\left.\begin{aligned} A_I &= \frac{a \cdot b \cdot \sin\beta}{2} \\ A_{II} &= \frac{c \cdot d \cdot \sin\delta}{2} \end{aligned}\right\} \oplus \qquad A = A_I + A_{II}$$



Bsp. 5.4.



$\alpha = 12{,}7°$

$\beta = 36{,}5°$

$s = 2\,km$

$$\frac{x}{\sin\alpha} = \frac{s}{\sin(\beta-\alpha)} \Rightarrow \boxed{x} = \frac{s\cdot\sin\alpha}{\sin(\beta-\alpha)}$$

$$\sin\beta = \frac{h}{x} \Rightarrow \boxed{h} = x\cdot\sin\beta$$

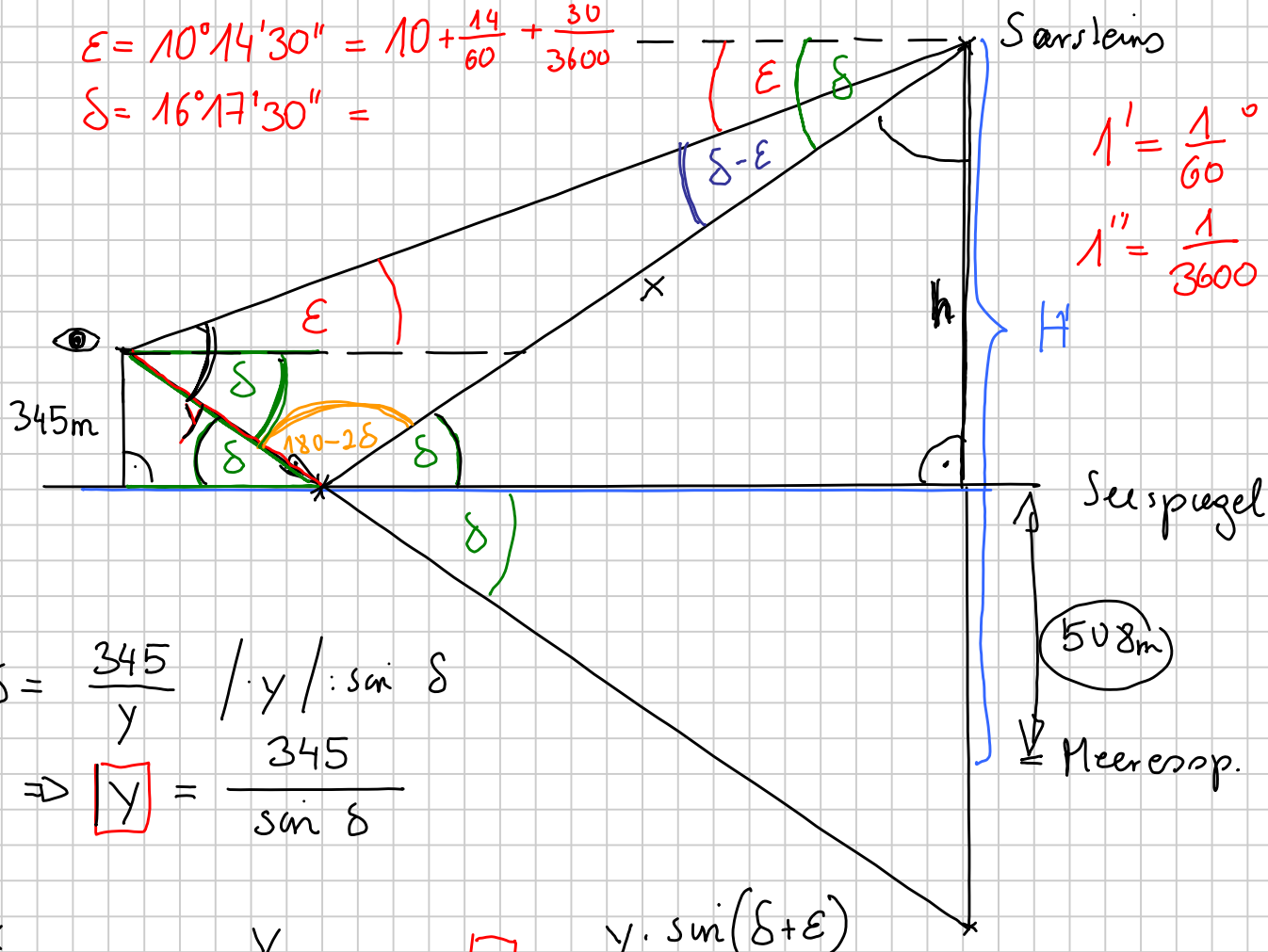
B.5.11

$\varepsilon = 10°14'30'' = 10 + \frac{14}{60} + \frac{30}{3600}$

$\delta = 16°17'30'' =$

$1' = \frac{1}{60}°$

$1'' = \frac{1}{3600}$



$$\sin\delta = \frac{345}{y} \quad |\cdot y\,|:\sin\delta$$

$$\Rightarrow \boxed{y} = \frac{345}{\sin\delta}$$

$$\frac{x}{\sin(\delta+\varepsilon)} = \frac{y}{\sin(\delta-\varepsilon)} \Rightarrow \boxed{x} = \frac{y\cdot\sin(\delta+\varepsilon)}{\sin(\delta-\varepsilon)}$$

$$\sin\delta = \frac{h}{x} \Rightarrow \boxed{h} = x\cdot\sin\delta$$

$$H = h + 508\,m = 1970{,}1\,m$$

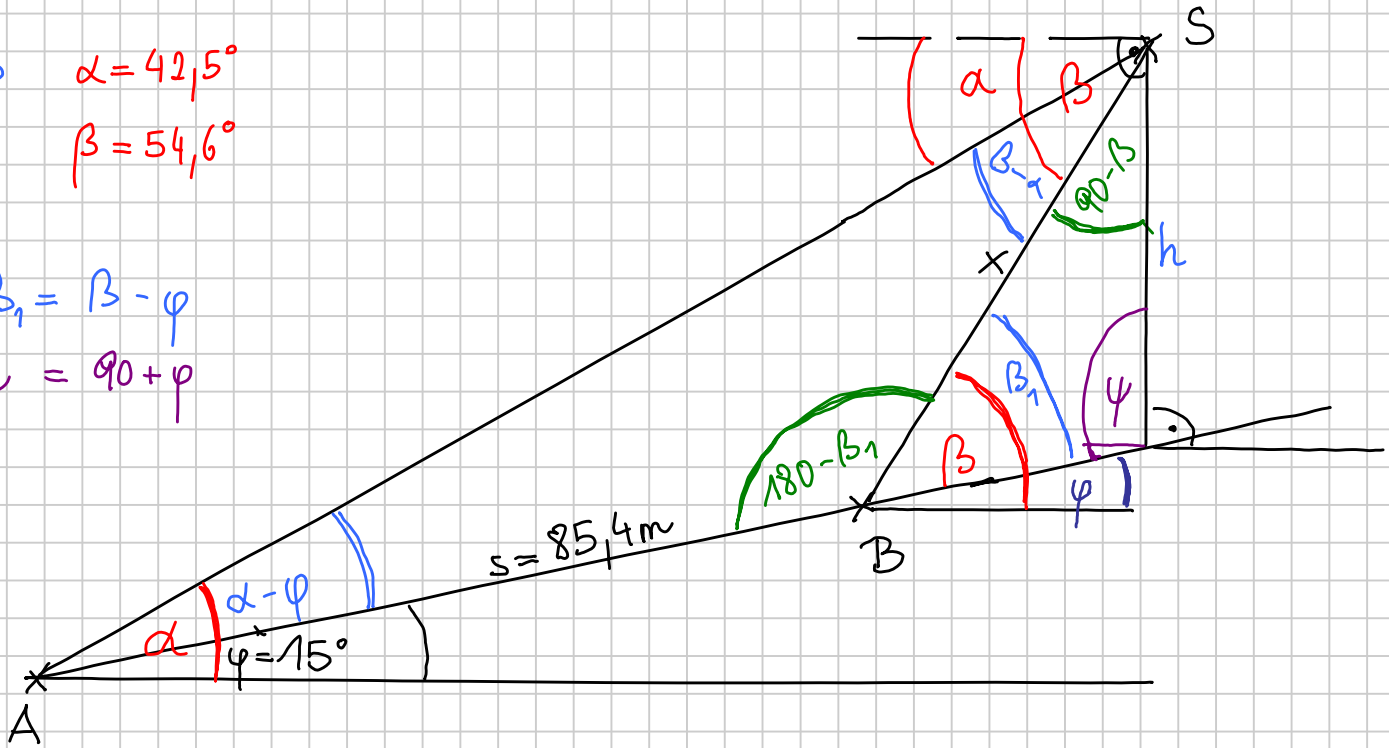
B.5.13

$\alpha = 42{,}5°$

$\beta = 54{,}6°$

$\beta_1 = \beta - \varphi$

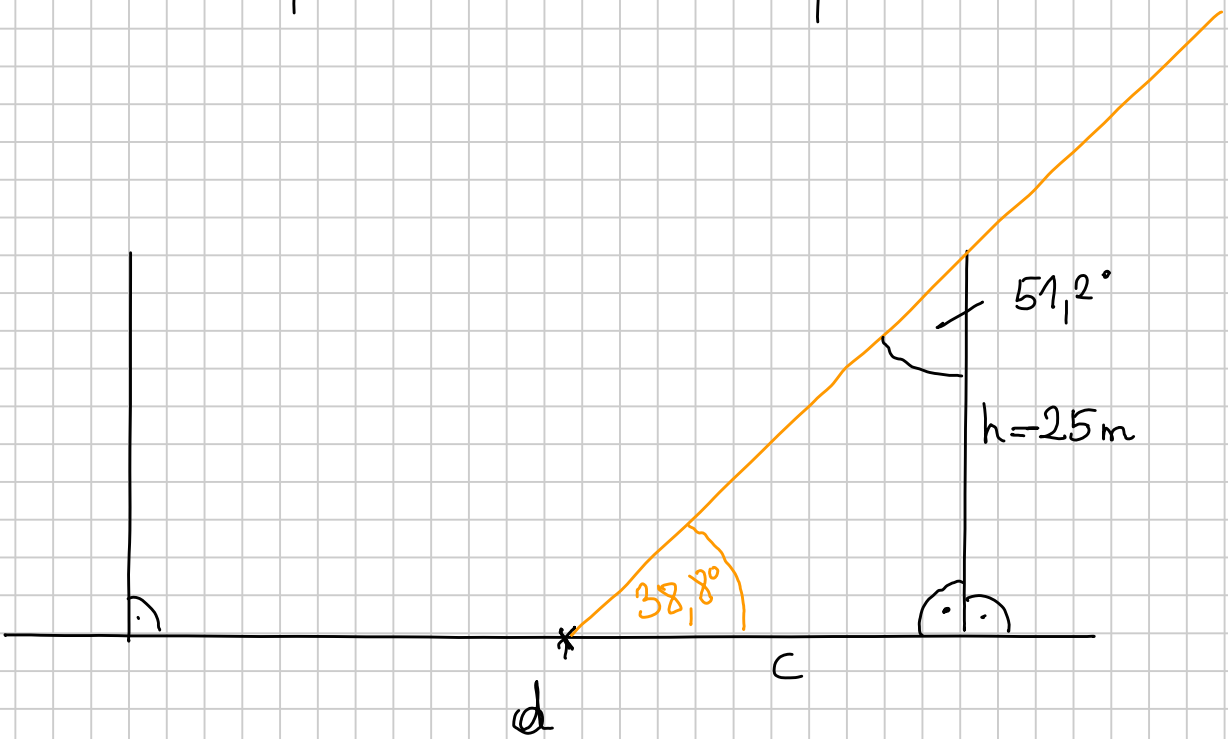
$\psi = 90 + \varphi$



$$\frac{x}{\sin(\alpha-\varphi)} = \frac{s}{\sin(\beta-\alpha)} \Rightarrow \boxed{x} = \frac{s \cdot \sin(\alpha-\varphi)}{\sin(\beta-\alpha)}$$

$$\frac{h}{\sin \beta_1} = \frac{x}{\sin \psi} \Rightarrow \boxed{h} = \frac{x \cdot \sin \beta_1}{\sin \psi}$$

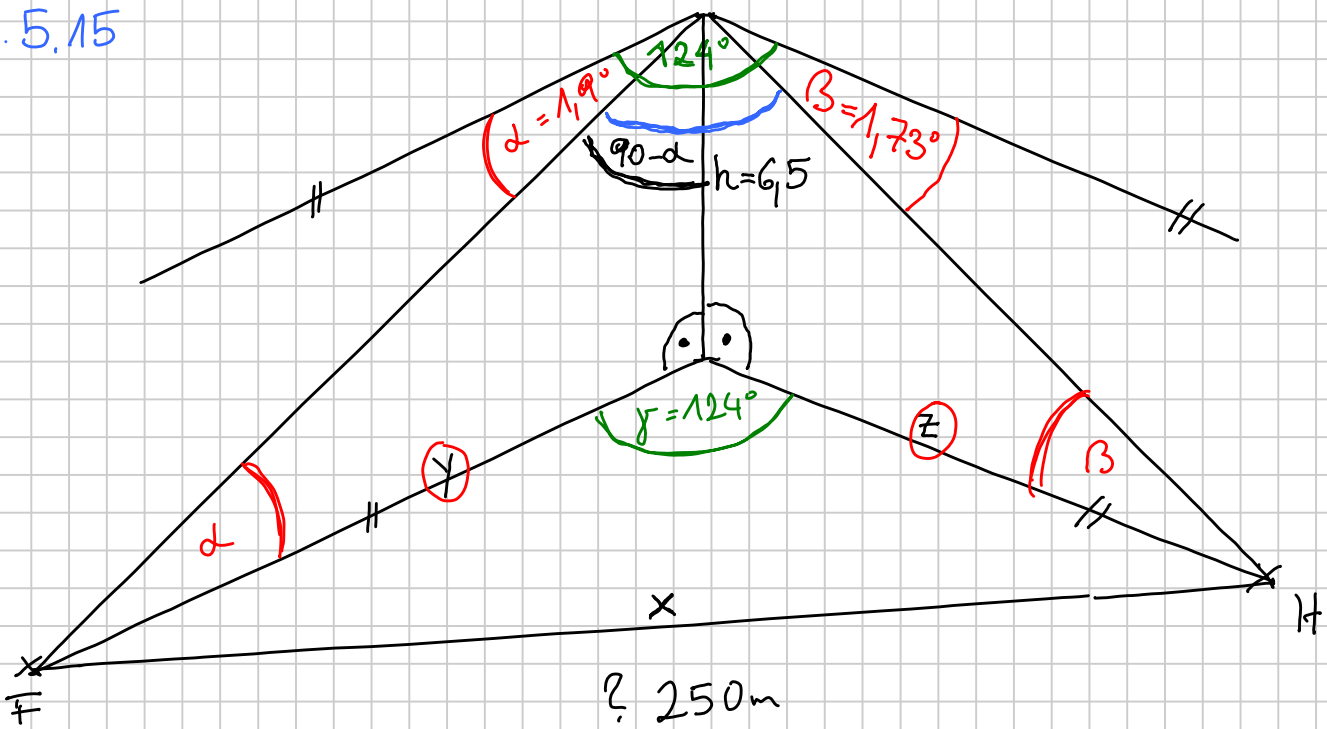
B.5.8.



$$\tan 38{,}8° = \frac{25}{c} \Rightarrow c = \frac{25}{\tan 38{,}8°} \qquad 2 \cdot c = d$$

$$\frac{c}{\sin 51{,}2} = \frac{25}{\sin 38{,}8°} \Rightarrow c = \frac{25 \cdot \sin 51{,}2}{\sin 38{,}8°}$$

B.5.15



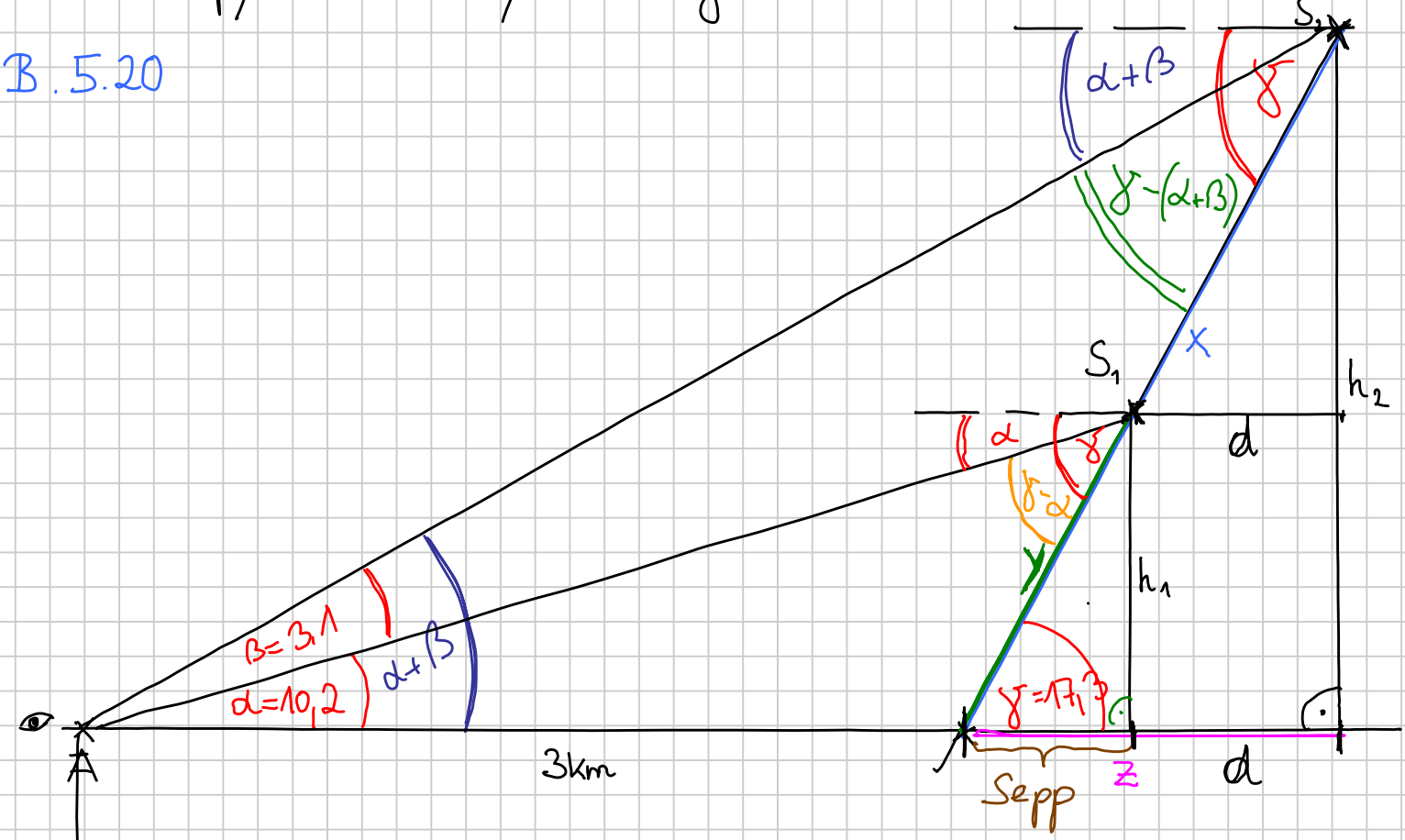
$$\frac{y}{\sin(90-\alpha)} = \frac{h}{\sin\alpha} \quad \Rightarrow \quad \boxed{y} = \frac{h \cdot \sin(90-\alpha)}{\sin\alpha}$$

$$\tan\beta = \frac{h}{z} \quad \Rightarrow \quad \boxed{z} = \frac{h}{\tan\beta}$$

$$x^2 = y^2 + z^2 - 2 \cdot y \cdot z \cdot \cos\gamma$$

$$x = \sqrt{y^2 + z^2 - 2 \cdot y \cdot z \cdot \cos\gamma}$$

B.5.20



+357m +1,4 = 358,4m

$$\frac{x}{\sin(\alpha+\beta)} = \frac{3000}{\sin(\gamma-(\alpha+\beta))} \quad \Rightarrow \quad \boxed{x} = \frac{3000 \cdot \sin(\alpha+\beta)}{\sin(\gamma-(\alpha+\beta))}$$

$$\sin\gamma = \frac{h_2}{x} \quad \Rightarrow \quad \boxed{h_2} = x \cdot \sin\gamma$$

$$\boxed{H_2} = h_2 + 358,4$$

$$\frac{y}{\sin\alpha} = \frac{3000}{\sin(\gamma-\alpha)} \quad \Rightarrow \quad \boxed{y} = \frac{3000 \cdot \sin\alpha}{\sin(\gamma-\alpha)}$$

$$\sin\gamma = \frac{h_1}{y} \quad \Rightarrow \quad \boxed{h_1} = y \cdot \sin\gamma$$

$$H_1 = h_1 + 358{,}4$$

$$z^2 = x^2 - h_2^2 \quad \Rightarrow \quad z = \sqrt{x^2 - h_2^2}$$

$$Sepp^2 = y^2 - h_1^2 \quad \Rightarrow \quad Sepp = \sqrt{y^2 - h_1^2}$$

$$d = z - Sepp = 5343\,m$$

$$0{,}107\ldots\,m \quad (: 50.000)$$

$$\underline{\underline{10{,}7\,cm}}$$