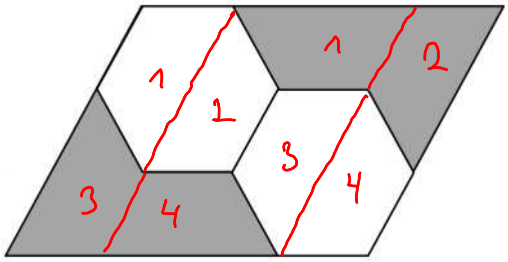
Integralrechnung

Denksport

16) In der Abbildung sind die beiden regelmäßigen Sechsecke kongruent. Welcher Bruchteil der Parallelogrammsfläche ist grau?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) $\frac{2}{5}$ E) $\frac{5}{12}$

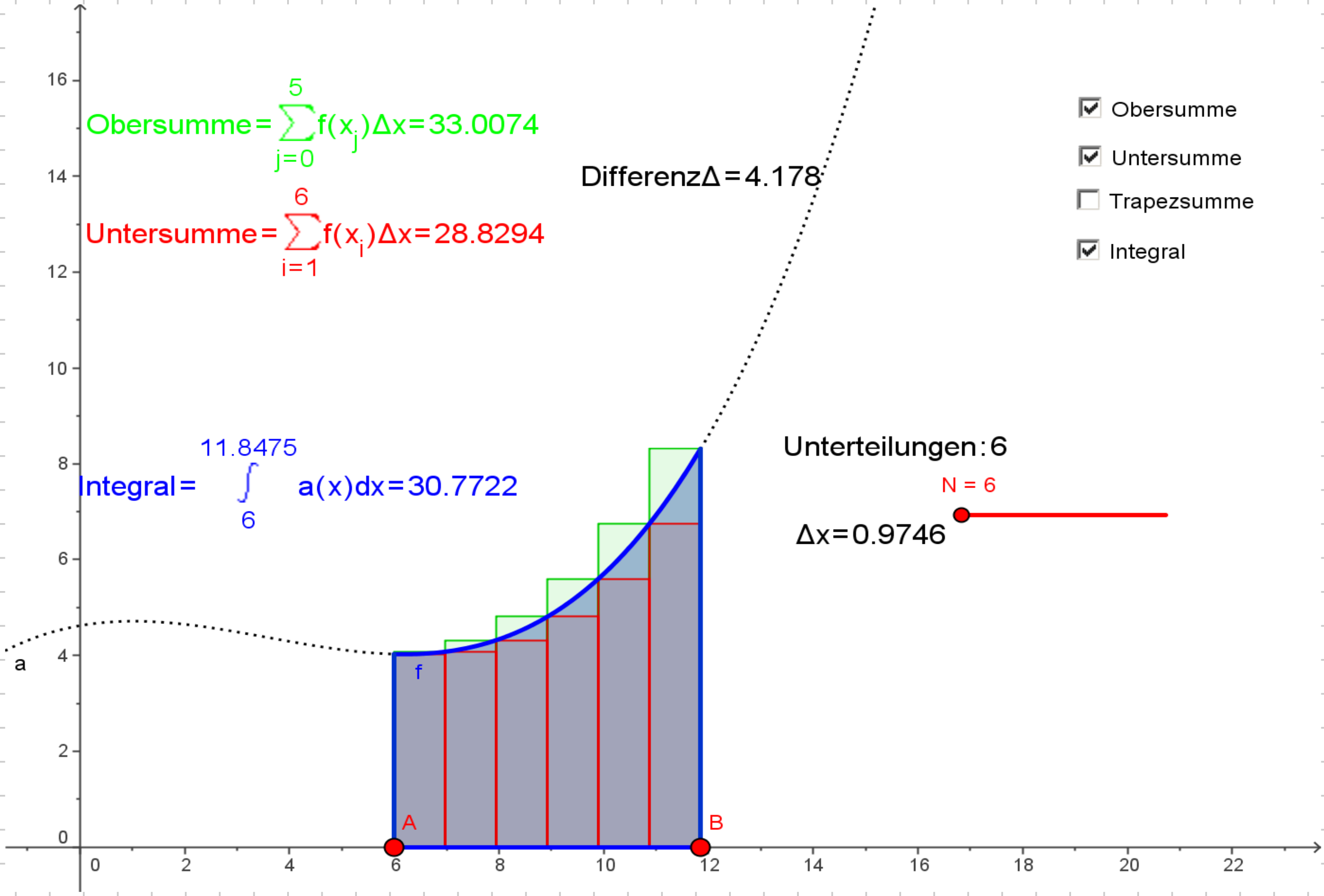


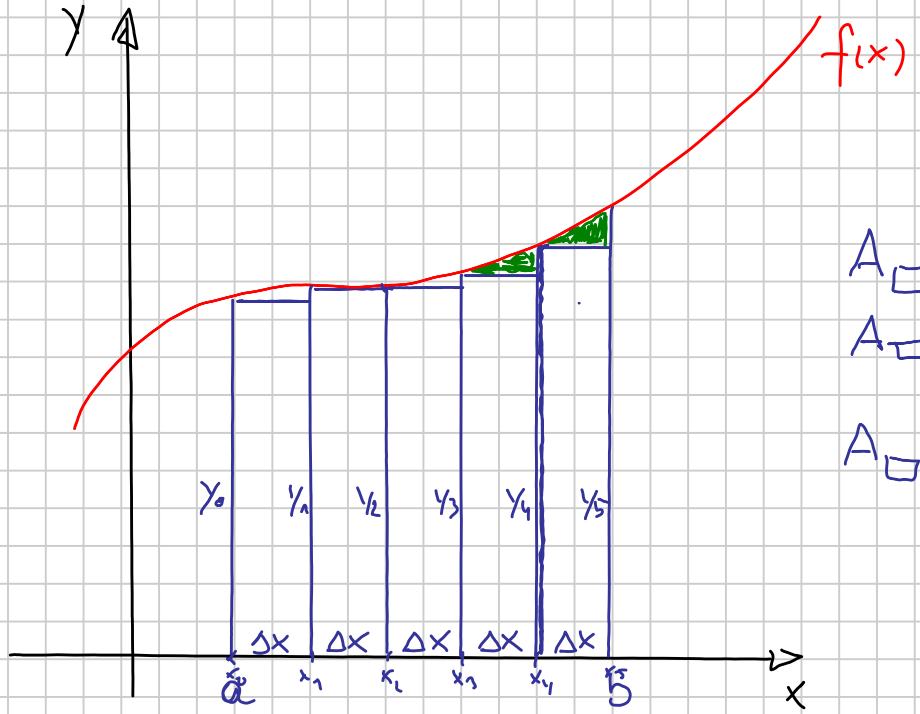
→ Termin : FR, 16.4.10 18^{00} - 22^{00} nicht vergessen

→ Termin (Ersatz)

Wiederholung

Grundidee der Integralrechnung Leibniz (1646 - 1716)





$A_{\square} = l \cdot b$

$A_{\square} = y_i \cdot \Delta x$

$A_{\square} = f(x_i) \cdot \Delta x$

$A = f(x_0) \cdot \Delta x + f(x_1) \cdot \Delta x + f(x_2) \cdot \Delta x + \ldots + f(x_4) \cdot \Delta x$

$A = \sum_{i=0}^{4} f(x_i) \cdot \Delta x$

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x = \int f(x) \cdot dx$$

Integralzeichen

$\Rightarrow$ Integrieren (~ Wurzelzeichen)
~ Ableiten

Ableiten

$f(x) = x^3 \qquad f'(x) = 3x^2$

Integrieren

Bsp. $\int x^2 \cdot dx$ „Integral von x^2 nach dx"

= Gesucht ist eine Fkt, deren Ableitung x^2 ist

$$\frac{x^3}{3} \xrightarrow{\text{Abl.}} \frac{\cancel{3}x^2}{\cancel{3}} = x^2$$

$$\int x^2 \cdot dx = \frac{x^3}{3} + C$$ const.

$\longrightarrow \frac{\cancel{3} x^2}{\cancel{3}} = x^2$

Bsp. $$\int x^3 \cdot dx = \frac{x^4}{4} + C$$

Bsp. $$\int x \cdot dx = \frac{x^2}{2} + C$$

$C \in \mathbb{R}$

$\frac{x^3}{3} + 4 \xrightarrow{\text{Abl.}} x^2$

$\frac{x^3}{3} - 5 \longrightarrow x^2$

$\frac{x^3}{3} + 10.000 \longrightarrow x^2$

Integrationsregeln

- $$\int x^n \cdot dx = \frac{x^{n+1}}{n+1} + C \qquad n \in \mathbb{R} \setminus \{-1\}$$

Bsp. $$\int x^7 \cdot dx = \frac{x^8}{8} + C$$

$$\int \frac{1}{x^3} \cdot dx = \int x^{-3} \cdot dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$$

$$\int \sqrt{x} \cdot dx = \int x^{\frac{1}{2}} \cdot dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2 \cdot \sqrt{x^3}}{3} + C$$

$$\int \frac{1}{x} \cdot dx = \int x^{-1} \cdot dx = \frac{x^0}{0} + C \quad ↯$$

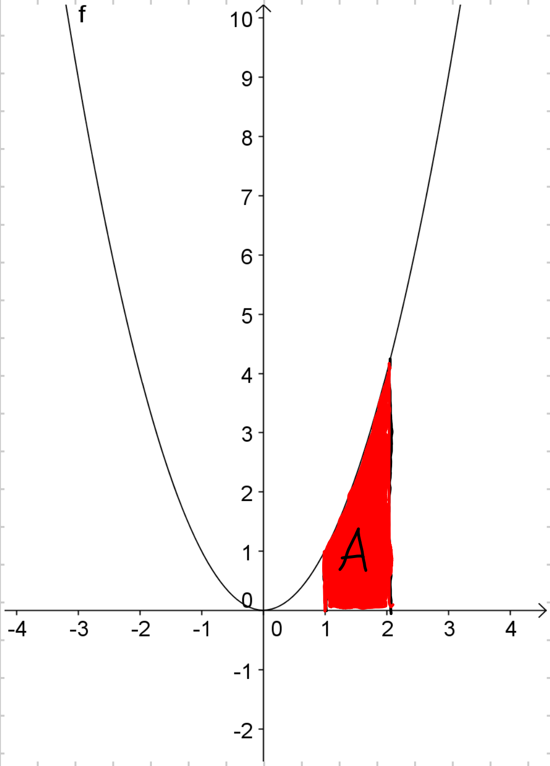
- $$\int \frac{1}{x} \cdot dx = \ln x + C$$
- $$\int e^x \cdot dx = e^x + C$$
- $$\int 1 \cdot dx = x + C$$
- $$\int 0 \cdot dx = C$$
- $$\int f(x) \pm g(x) \cdot dx = \int f(x) \cdot dx \pm \int g(x) \cdot dx$$

$$\int x^3 + 7x^2 \cdot dx = \frac{x^4}{4} + 7 \cdot \frac{x^3}{3} + C$$

Unbestimmtes Integral (Funktion)

Unbestimmtes - Bestimmtes Integral

Bsp: $f(x) = x^2$



Bestimmtes Integral (Zahl)

Flächenberechnung

$$A = \int_1^2 x^2 \cdot dx =$$

$$= \frac{x^3}{3} + C \Big|_1^2 =$$

$$= \left(\frac{2^3}{3} + C\right) - \left(\frac{1^3}{3} + C\right)$$

$$= \frac{8}{3} + \cancel{C} - \frac{1}{3} - \cancel{C}$$

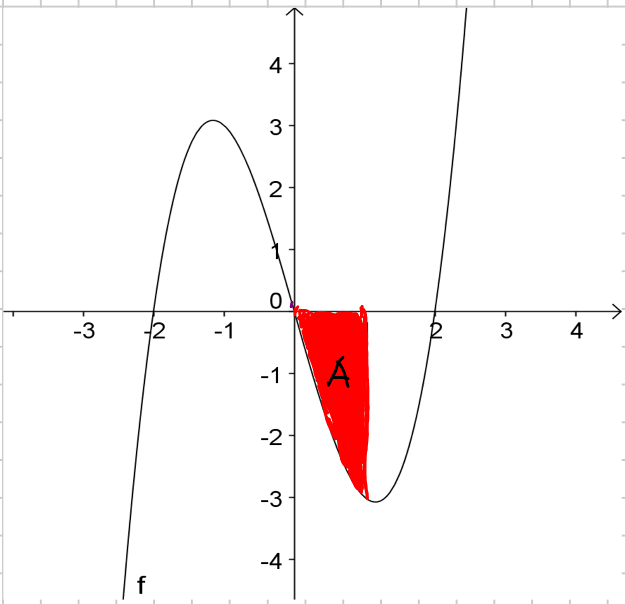
$$\int f(x)\, dx = F(x) \quad \text{Stammfunktion} \qquad = \underline{\frac{7}{3}\ cm^2}$$

FE Flächeneinheit E^2

$$\text{Allg: } \quad A = \int_a^b f(x) \cdot dx = F(x) \Big|_a^b = F(b) - F(a)$$

Bsp $\quad f(x) = x^3 - 4x$

? Fläche zwischen Funktion und x-Achse im Intervall $[0; 1]$?



$$A = \left| \int_0^1 x^3 - 4x \cdot dx \right| = \left| \frac{x^4}{4} - \cancel{4}^{2} \cdot \frac{x^2}{\cancel{2}} \Big|_0^1 \right|$$

$$= \left| \left(\frac{1^4}{4} - 2 \cdot 1^2\right) - \cancel{\left(\frac{0^4}{4} - 2 \cdot 0^2\right)} \right| \quad 0$$

$$= \left| \frac{1}{4} - 2 \right| = \left| -\frac{7}{4} \right| = \left| -1{,}75 \right|$$

$$\text{Fläche} = |-1{,}75| = +1{,}75 E^2$$

Flächenberechnung

Bsp. Berechnen

Berechnen Sie die Fläche, die die Funktion f(x) mit der x-Achse einschließt.

$f(x) = 0.5\ (x^3 - 2\ x^2 - 3\ x)$

(1) Grenzen berechnen! $\Rightarrow$ Nullstellen

$$\cancel{0{,}5} \cdot (x^3 - 2x^2 - 3x) \overset{!}{=} 0 \quad |:0{,}5$$

$$x \cdot (x^2 - 2x - 3) = 0$$

$x = 0$ oder $x^2 - 2x - 3 = 0$

$N_1(0|0)$ $\quad$ ↳ ABC-Formel TI82 $\quad A = 1$, $B = -2$, $C = -3$

$\Rightarrow x_2 = 3 \quad N(3|0)$

$\Rightarrow x_3 = -1 \quad N(-1|0)$

$$A = \int_{-1}^{0} \frac{1}{2} \cdot (x^3 - 2x^2 - 3x) \cdot dx + \left| \int_{0}^{3} \frac{1}{2} (x^3 - 2x^2 - 3x) \cdot dx \right| =$$

$$= \frac{1}{2} \cdot \left(\frac{x^4}{4} - 2 \cdot \frac{x^3}{3} - 3\frac{x^2}{2} \right) \Bigg|_{-1}^{0} + \left| \frac{1}{2} \cdot \left(\frac{x^4}{4} - 2 \cdot \frac{x^3}{3} - 3\frac{x^2}{2} \right) \Bigg|_{0}^{3} \right| =$$

$$= \frac{1}{2} \left[(0) - \left(\frac{(-1)^4}{4} - 2 \cdot \frac{(-1)^3}{3} - 3 \cdot \frac{(-1)^2}{2} \right) \right]$$

$$+ \left| \frac{1}{2} \cdot \left[\left(\frac{3^4}{4} - 2 \cdot \frac{\cancel{3}^{3}\,2}{\cancel{3}} - 3 \cdot \frac{3^2}{2} \right) - 0 \right] \right|$$

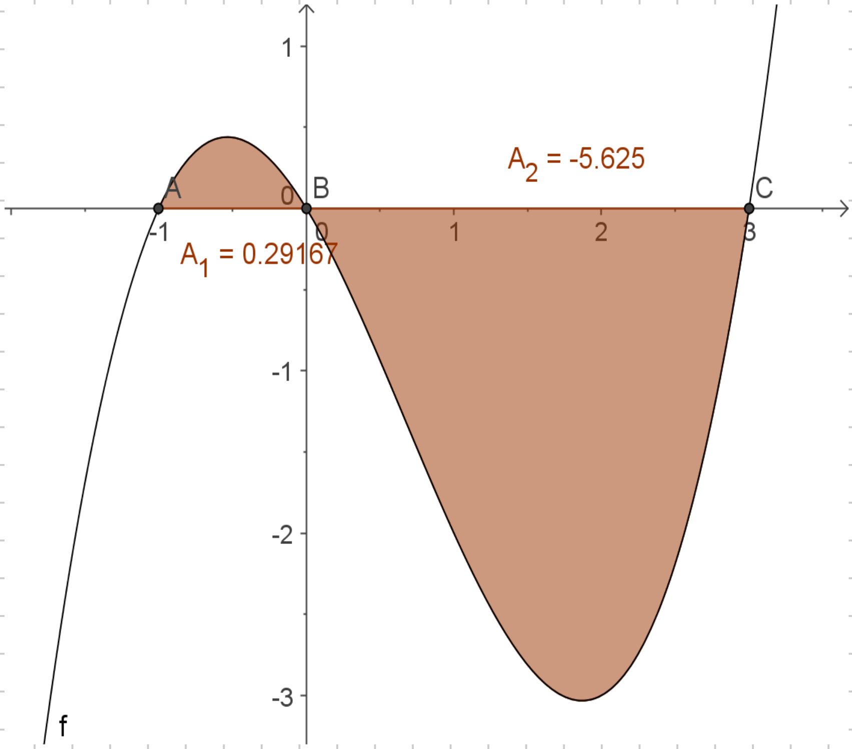
$$= \frac{1}{2} \left[0 - \left(\frac{1}{4} + \frac{2}{3} - \frac{3}{2} \right) \right] + \left| \frac{1}{2} \left[\frac{81}{4} - 18 - \frac{27}{2} \right] \right| =$$

$3 + 8 - 18$ $\qquad$ $81 - 72 - 54$

$$= \frac{1}{\cancel{2}} \cdot \left[-\left(-\frac{7}{12} \right) \right] + \left| \frac{1}{2} \quad \frac{-63}{4} \right|$$

$$\frac{7}{24} + \left| -\frac{45}{8} \right| = \frac{142}{24}$$

$$0{,}2916 + \left| -5{,}625 \right| = \underline{5{,}917\ E^2}$$

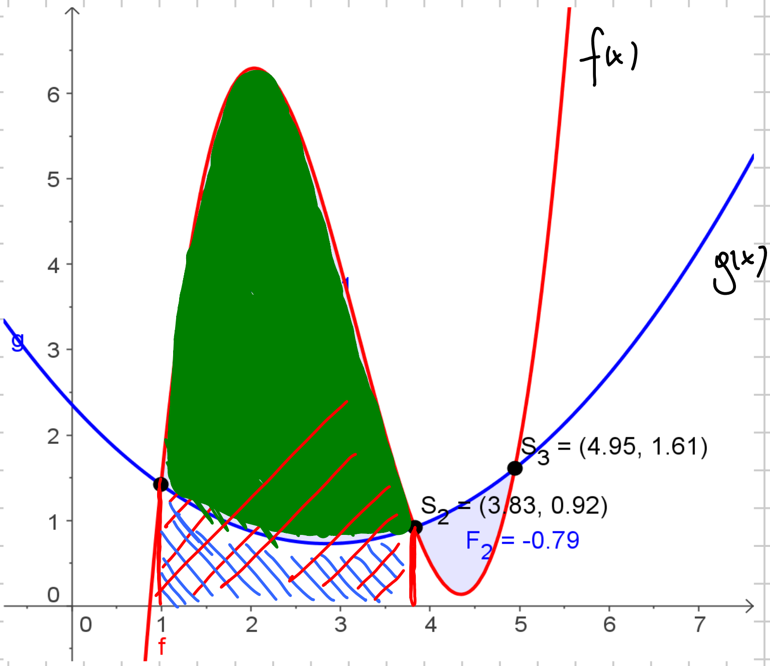


T182 :

$$\boxed{\mathrm{fnInt}\left(\tfrac{1}{2}\cdot(x^3-2x^2-3x),\ x,\ -1,\ 0\right) = 0{,}29\ldots}$$

$$\mathrm{fnInt}\left(f(x),\ x,\ UG,\ OG\right)$$

Fläche zwischen zwei Funktionen



$A_{rot} - A_{blau}$

$$\int f(x)\cdot dx - \int g(x)\cdot d(x)$$

$$A = \int_a^b f(x) - g(x)\ \cdot dx$$

Bsp. $f(x) =$
$g(x) =$ $\Big\} \Rightarrow$

$$9 - x^2 = 2x + 6 \quad | + x^2 \quad | - 9$$
$$0 = x^2 + 2x - 3$$

$\Rightarrow$ TI82 ABC - Formel $A = 1$, $B = 2$, $C = -3$

$x_1 = -3$

$x_2 = 1$

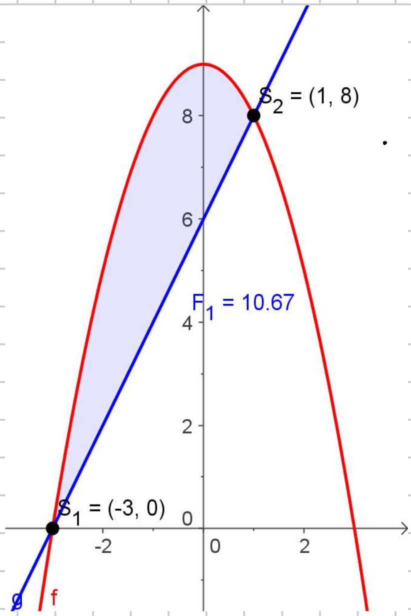
$$\Rightarrow A = \int_{-3}^{1} (9 - x^2) - (2x + 6) \cdot dx = \int_{-3}^{1} -x^2 - 2x + 3 \cdot dx =$$

$$= -\frac{x^3}{3} - \cancel{2} \cdot \frac{x^2}{\cancel{2}} + 3x \Big|_{-3}^{1} =$$

$$\left(-\frac{1^3}{3} - 1^2 + 3 \cdot 1\right) - \left(-\frac{(-3)^3}{3} - (-3)^2 + 3 \cdot (-3)\right) =$$

$$\left(-\frac{1}{3} - 1 + 3\right) - (+9 - 9 - 9) =$$

$$\frac{5}{3} + 9 = \frac{32}{3} = 10\frac{2}{3} \; E^2$$



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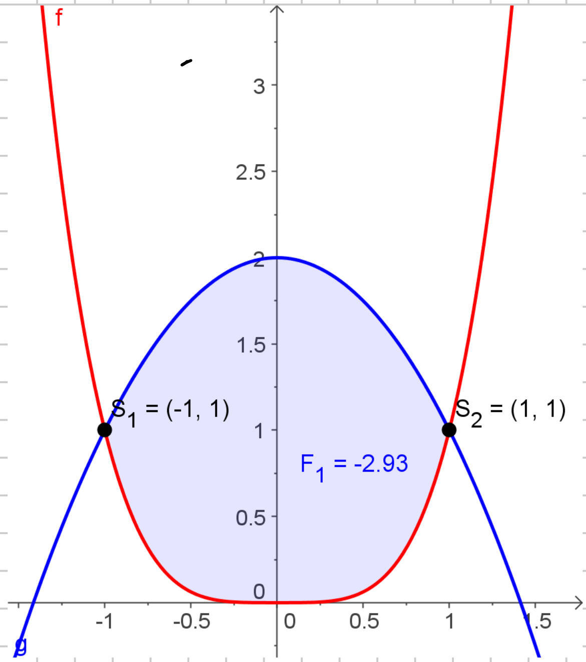
$f(x) = x^4$

$g(x) = 2 - x^2$

$$x^4 = 2 - x^2$$
$$x^4 + x^2 - 2 = 0$$

$u = x^2$



$$u^2 + u - 2 = 0$$

$$\Rightarrow \quad u_1 = -2 = x^2 \rightarrow \text{keine reele Zahl}$$

$$u_2 = 1 = x^2 \rightarrow x_1 = -1, \quad x_2 = +1$$

$$A = \int_{-1}^{1} 2 - x^2 - x^4 \cdot dx = 2x - \frac{x^3}{3} - \frac{x^5}{5} \Big|_{-1}^{1} =$$

$$\text{fnInt}(2 - x^2 - x^4, x, -1, 1) = 2{,}93\ldots E^2$$

Bsp. 35

f(x) = x³ - 4 x² + x + 5
g(x) = x² -4

$$x^3 - 4x^2 + x + 5 = x^2 - 4$$

$$x^3 - 5x^2 + x + 9 = 0$$

$\Rightarrow$ SOLVER TI82 :

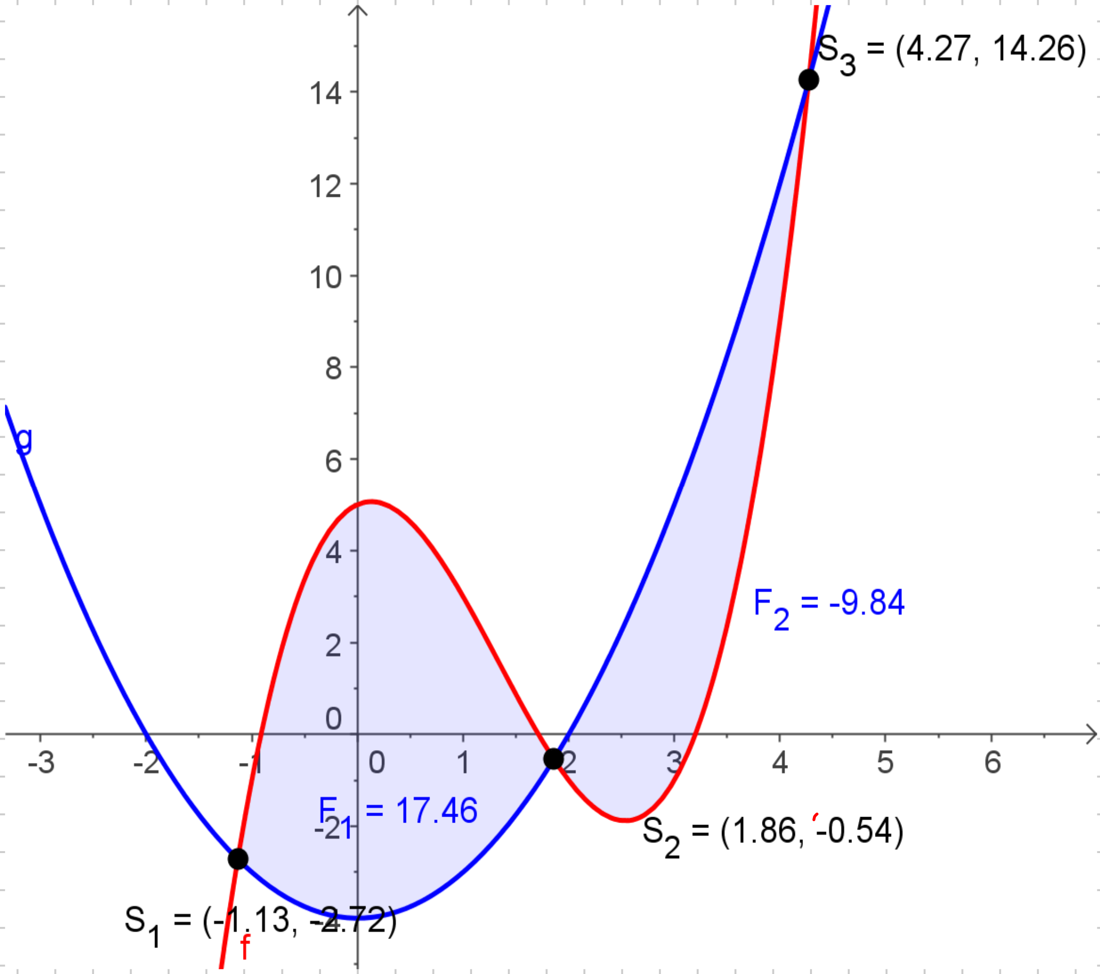
Startwert : 0 $\Rightarrow x_1 = -1{,}13$ $\boxed{A}$
Startwert : 5 $\Rightarrow x_2 = 4{,}27$ $\boxed{B}$
Startwert : 1 $\Rightarrow x_3 = 1{,}86$ $\boxed{C}$

$$A = \int_{-1{,}13}^{1{,}86} x^3 - 4x^2 + x + 5 - (x^2 - 4) \cdot dx + \left| \int_{1{,}86}^{4{,}27} x^3 - 4x^2 + x + 5 - (x^2 - 4) \cdot dx \right|$$

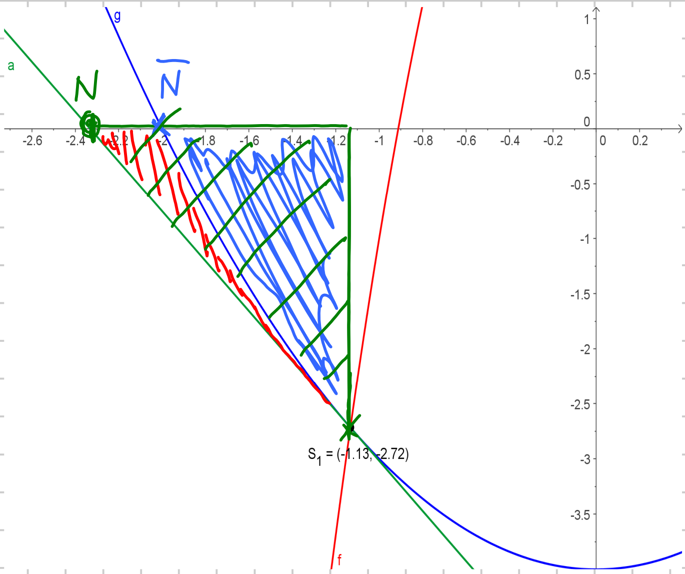
$$A = \text{fnInt}(x^3 - 5x^2 + x + 9, x, \boxed{A}, \boxed{C}) + \left| \text{fnInt}(x^3 - 5x^2 + x + 9, x, \boxed{C}, \boxed{B}) \right|$$

$$= 17{,}46 + |-9{,}84|$$

$$= 27{,}295 \approx 27{,}3 \; E^2$$



Ausblick:



$$\int_{N}^{S_1} \text{tangente} - \int_{\bar{N}}^{S_1} g(x) \cdot dx$$